Math 29
Worksheet 9
Economics

The Gross Domestic Product (or GDP) is defined as the total market value of all "finished goods" and services produced in the country during a specific time period, normally a year. By "finished goods" economists mean goods that are in their final state, rather than those that will be modified and then resold. For example, a pair of jeans bought by an individual is in its final state, whereas a bolt of cloth bought by a clothing manufacturer is not in its final state. If the GDP increases from one year to another by roughly $2.5 - 3.5\%$, the economy is considered healthy. If it decreases two periods in a row, then the economy is said to be in a recession.

While the GDP is defined in terms of goods and services produced, it is computed as the total expenditures of the country on finished goods and services in a year. This total is the sum of household expenditures, business expenditures, government expenditures, and foreign expenditures. More specifically, the formula for total expenditures is

$$E = C + I + G + Nx$$

where the variables are defined as follows.

- C (for *consumption*) denotes the total expenditures of households on goods and services.

- I (for *investment*) denotes the total expenditures of businesses on factories, equipment, and technology.

- G denotes the total expenditures of government on goods and services.

- Nx denotes the total monetary value of all finished goods that are exported, minus the total monetary value of all finished goods that are imported.

The variable C is broken down by the equation

$$C = a + b(Y - T)$$

where a represents the minimum amount that is spent by all households on necessities (like rent), Y represents the total income of all households, T represents the total amount of taxes paid by all households, and b represents the average percent of disposable income that households spend on goods and services. The idea of this equation is that you have to spend a, and then households decide on some percent of their income after taxes to spend on non-necessities.

The variable I is broken down by the equation

$$I = F - kr$$

where F is the amount that businesses invest in factories, equipment, and technology, r is the current interest rate, and k is a constant. This equation tells us that the higher the interest rate is, the less money businesses are willing to borrow money to invest in factories, equipment, and technology.

The Gross Domestic Product GDP is said to be in a *state of equilibrium* if the total expenditures of the economy $E = C + I + G + Nx$ is equal to the total income of the economy Y. That is, if $E = Y$.

1. Consider a fictitious economy in which government purchases, taxes, and net exports are all zero, and where $a = 300$, $b = .75$, $F = 1000$, and $k = 100$. Draw graphs of total expenditures E as a function of total income Y (this means Y is the x-axis and E is the y-axis) when the interest rate is 2%, 5% and 10%. Then for each of these interest rates determine what value of Y will cause the GDP to be in a state of equilibrium (this means when do the graphs cross the line $y = x$).

2. Assume the GDP is in a state of equilibrium, and we have the following equation

$$E = \alpha + \beta(Y - \theta - tY) + \lambda - \delta Y$$

where the parameters α, β, θ, λ, δ, $t > 0$, $\beta(1 - t) < 1$, and $\alpha > \beta\theta$. Solve for Y in terms of the parameters of the model.

For the problems below you should assume that every month has precisely 30 days and a year has 360 days.

3. You have a credit card bill of $7200. In order to avoid having the interest build on itself, every month you pay off $600 of your debt plus all the interest that you owe up to that point. After making these payments for 12 months you have paid off your debt. Your credit card company charges 6.7% annual interest compounded daily. At the end of the 12 months what is the total amount of interest that you have paid?

4. You have a credit card debt of $4,800.00 with a credit card company that charges 18.1% annual interest rate compounded monthly. You receive an offer to transfer your debt to another credit card company which charges only 17.90% annual interest rate compounded daily. However, there is a $100 fee for transferring the debt. Whether or not you transfer the debt, you plan to pay $100 every month plus all of the interest you owe up to that point. Is it worth it to transfer your debt to the credit card with the lower interest rate?

5. Christina has a credit card bill of A. Her credit card company charges 10% compounded daily. Each month she pays $\frac{A}{10}$ together with all of the interest she owes up until that point. The total interest she pays over the time that it takes her to pay off her debt is $1000. How much is A?

6. Suppose you have a credit card debt of P with the credit card company that charges 5.99% annual interest rate compounded daily. You receive an offer from another credit card company for a 3.70% annual interest rate compounded daily on balance transfers for six months, however there is a transaction fee of $40 for the balance transfer. You plan to pay off your debt in six months by paying the same amount each month, together with all the interest that you owe up to that point. How large would your original balance P have to be so that the balance transfer is to your advantage?