Math 152 – Fall 2006
Professor Hardin
Exam #1

Name: ______________________

1. Treat the data in the table below as a random sample of size 16 from a Poisson distribution with mean θ. (Note these are real data from how many touchdowns Peyton Manning threw in 16 consecutive games, so they aren't really a random sample.)

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
X_i	2	2	5	2	3	3	5	4	5	4	6	3	2	1	2	0

   (a) (+5 points) Find the MLE for θ.

   (b) (+15 points) Let X_{17} be the next independent observation. Find **and INTERPRET** an asymptotic 98% confidence interval for $P(X_{17} = 0)$. (Hint: you will need to find Fisher's information to complete this problem.)

2. Let X_1, X_2 be independently distributed $N(\theta, \sigma^2 = 0.81)$. We'd like to test:

$$H_o : \theta = 4$$
$$H_1 : \theta = 5$$

We have two competing tests / critical regions:

$$\begin{aligned} C_1 &= \{(X_1, X_2) | X_1 > 5.48\} \\ C_2 &= \{(X_1, X_2) | X_1 + X_2 > k\} \end{aligned}$$

(a) (+10 points) Find k so that the size of C_2 is the same as the size of C_1.

(b) (+10 points) Which test is more powerful? (Numerically justify your conclusion.)

3. Let $X_1, X_2, X_3 \sim U[\theta, 2\theta], \theta > 0$. Note:

$$W = \min(X_i) \quad E(W) = \tfrac{5\theta}{4} \quad E(W^2) = \frac{8\theta^2}{5}$$
$$Y = \max(X_i) \quad E(Y) = \tfrac{7\theta}{4} \quad E(Y^2) = \frac{31\theta^2}{10}$$

(a) (+5 points) Find the MLE of θ.

(b) (+5 points) Find the Method of Moments (MOM) estimate of θ.

(c) (+10 points) Which estimator has a smaller MSE (Mean Squared Error)? Which estimator would you be more likely to use?

4. Suppose you know that your data have a beta distribution with $\alpha = 1$. However, you are unsure about the β parameter. Your test of interest is:

$$H_o : \beta \leq 5$$
$$H_1 : \beta > 5$$

Your data are as follows:

$$n = 50 \qquad \textstyle\sum_i X_i = 7.673 \qquad \displaystyle\prod_i X_i = 2.313 \times 10^{-50}$$

$$\textstyle\sum_i (1 - X_i) = 42.327 \qquad \displaystyle\prod_i (1 - X_i) = 0.0001379 = 1.379 \times 10^{-4}$$

(a) (+5 points) Find the MLE of β. (Here you can find the actual point estimate as well as the MLE as a function of the Xs.)

(b) (+10 points) Carry out a likelihood ratio test for the above hypotheses. You must derive the test and then make a decision about your hypotheses based on your data.

(c) (+5 points) Find the p-value of the test.

5. Let $X_1, X_2, \ldots, X_{10}$ be a random sample of size 10 from a distribution with pdf:

$$f(x;\theta) = e^{-(x-\theta)} \qquad \theta \le x < \infty$$

(a) (+10 points) **DERIVE** the pdf of Y_1 (the minimum value.)

(b) (+10 points) Compute $P(\theta \le Y_1 \le \theta + c)$ and use it to construct a 90% confidence interval for θ.