Math 152 – Fall 2007
Professor Hardin
Exam #1

Name: _____

You are allowed one sheet of notes, the distribution handout, and a calculator. Please do as much work on each problem as possible. Do not get stuck working on one problem, you will not get extra time! There will be lots of partial credit. Also, please sit one person to a table.

1. (+20 points) Suppose that $X_1, X_2, \ldots, X_n$ form a random sample from a Bernoulli distribution for which the parameter p is unknown. Show that the variance of every unbiased estimator of $(1-p)^2$ must be at least $4p(1-p)^3/n$. (Aside: this is interesting because we might, for example, want to estimate the probability of two failures in a row.)

2. (+20 points) Suppose we take a random sample, $X_1, X_2, X_3, X_4 \sim N(0, \sigma^2)$. Show that the distribution of $Y = \frac{X_1+X_2}{X_3-X_4}$ is t_1. Justify your conclusion using the results we learned in class.

3. (+20 points) Suppose that a random sample $X_1, X_2, \ldots, X_n$ is to be taken from a normal distribution with unknown mean (θ) and variance 100, and the prior distribution of θ is normal with a specified mean, μ, and a variance 25.

   (a) Find the posterior distribution of θ.

   (b) Suppose that θ will be estimated using squared error loss, but that there is a cost of 0.25 for each sampled observation. If the total cost of the estimation procedure is equal to the Bayesian MSE of the Bayes estimator plus the overall sampling cost (= 0.25n), **what is the sample size n for which the total cost will be a minimum**?

   (Note: the first step is to find the Bayesian MSE for the Bayes estimator of θ.)

Hint: you might need the chain rule for derivatives, for example, $\frac{\partial}{\partial y}(\frac{c}{f(y)}) = \frac{-cf'(y)}{(f(y))^2}$

4. (+20 points) Suppose we test three independent batteries; a rechargeable battery, a lithium battery, and an alkaline battery. The lifetime of the rechargeable battery (X_1) has an exponential distribution with mean 3λ; the lifetime of the lithium battery (X_2) has an exponential distribution with mean 2λ; and the lifetime of the alkaline battery (X_3) has an exponential distribution with mean λ. Find the MLE of λ based on the three observations X_1, X_2, X_3.

5. (a) (+10 points) Five independent samples, each of size n are to be drawn from a normal distribution where σ is known. For each sample, the interval $(\overline{x} - 0.96\frac{\sigma}{\sqrt{n}}, \overline{x} + 1.06\frac{\sigma}{\sqrt{n}})$ will be constructed. What is the probability that at least four of the intervals will contain the unknown μ?

(b) (+10 points) In what type of situation might a statistician use a posterior interval with an improper prior to construct an interval estimate (instead of a confidence interval)? Give at least 2 reasons for full credit.