

1.

$$\begin{aligned} f(x|p) &= p^x(1-p)^{1-x} \\ \lambda(x|p) &= x \ln p + (1-x) \ln(1-p) \\ \lambda'(x|p) &= \frac{x}{p} - \frac{1-x}{1-p} \\ \lambda''(x|p) &= \frac{-x}{p^2} - \frac{1-x}{(1-p)^2} \end{aligned}$$

$$\begin{aligned} I(p) = -E(\lambda''(x|p)) &= -[(1/p^2)E(-X) - (1/(1-p)^2)E(1-X)] \\ &= -[(1/p^2)(-p) - (1/(1-p)^2)(1-p)] \\ &= \frac{1}{p} + \frac{1}{(1-p)} \\ &= \frac{(1-p) + p}{p(1-p)} \\ &= \frac{1}{p(1-p)} \end{aligned}$$

$$\begin{aligned} m(p) &= (1-p)^2 = E(T) \text{ where } T \text{ estimates } (1-p)^2 \\ m'(p) &= -2(1-p) \\ (m'(p))^2 &= 4(1-p)^2 \end{aligned}$$

$$\begin{aligned} \text{CRLB} &= \frac{(m'(p))^2}{nI(p)} = \frac{4(1-p)^2}{n(1/(p(1-p)))} \\ &= \frac{4p(1-p)^3}{n} \end{aligned}$$

Any estimator which is unbiased for $(1-p)^2$ will have a variance at least as big as the relevant CRLB (above).

2. $\frac{X_1+X_2}{X_3-X_4}$ is a quotient..., $t_\nu = \frac{Z}{\sqrt{\chi_\nu^2/\nu}}$ is a quotient...

$$X_1 + X_2 \sim N(0, 2\sigma^2) \rightarrow \frac{X_1+X_2}{\sqrt{2\sigma^2}} \sim N(0, 1)$$

$$X_3 - X_4 \sim N(0, 2\sigma^2) \rightarrow \frac{X_3-X_4}{\sqrt{2\sigma^2}} \sim N(0, 1) \rightarrow \left(\frac{X_3-X_4}{\sqrt{2\sigma^2}}\right)^2 \sim \chi_1^2$$

$$\frac{\frac{X_1+X_2}{\sqrt{2\sigma^2}}}{\sqrt{\left(\frac{X_3-X_4}{\sqrt{2\sigma^2}}\right)^2/1}} = \frac{\frac{X_1+X_2}{\sqrt{2\sigma^2}}}{\frac{X_3-X_4}{\sqrt{2\sigma^2}}} = \frac{X_1 + X_2}{X_3 - X_4} \sim t_1$$

3. (a) We know: $\theta|\underline{X} \sim N\left(\frac{\sigma^2\mu+n\nu^2\bar{X}}{\sigma^2+n\nu^2}, \frac{\sigma^2\nu^2}{\sigma^2+n\nu^2}\right) = N\left(\frac{100\mu+n25\bar{X}}{100+25n}, \frac{2500}{100+25n}\right)$

(b) The Bayes MSE = $E[(\hat{\theta} - \theta)^2|\underline{X}] = Var(\theta|\underline{X}) + \{E[(\hat{\theta} - \theta)|\underline{X}]\}^2$, but because $\hat{\theta} = E[\theta|\underline{X}]$, $\hat{\theta}$ is unbiased, and the Bayes MSE = $Var(\theta|\underline{X})$.

$$\text{total cost} = \text{TC} = Var(\theta|\underline{X}) + 0.25n = \frac{\sigma^2\nu^2}{\sigma^2+n\nu^2} + 0.25n$$

$$\frac{\partial \text{TC}}{\partial n} = \frac{-\sigma^2\nu^2\nu^2}{(\sigma^2+n\nu^2)^2} + 0.25 = 0$$

$$n=16$$

4. $f(\underline{x}) = \frac{1}{3\lambda} e^{-x_1/3\lambda} \frac{1}{2\lambda} e^{-x_2/2\lambda} \frac{1}{\lambda} e^{-x_3/\lambda}$

$$L(\lambda) = \ln(f(\underline{x})) = \ln(1/3) + \ln(1/2) + \ln(1/\lambda^3) - \frac{1}{\lambda}\left(\frac{x_1}{3} + \frac{x_2}{2} + x_3\right)$$

$$\frac{\partial L(\lambda)}{\partial \lambda} = \frac{-3}{\lambda} + \frac{(\frac{x_1}{3} + \frac{x_2}{2} + x_3)}{\lambda^2} = 0$$

$$\hat{\lambda} = \frac{x_1}{9} + \frac{x_2}{6} + \frac{x_3}{3}$$

5. (a) The probability that the given interval will contain μ is $P(-0.96 \leq Z \leq 1.06) = 0.8554 - 0.1685 = 0.6869$. The probability that 4 out of 5 such intervals contains μ is binomial with $n = 5$ and $p = 0.6869$. So, the probability is $5 \cdot 0.6869^4(1 - 0.6869) + 0.6869^5 = 0.501$.

(b) i. The statistician is a Bayesian and wants to interpret the interval using a probabilistic statement (and wants to consider the parameters as random variables)...

ii. BUT, she doesn't have any real prior information (or doesn't believe the prior information she does have.)