Math 152 – Fall 2006
Professor Hardin
Exam #2
(out of 85 points)

Name: ________________________

1. Suppose that $X_1, X_2, \ldots, X_n$ are a random sample from an exponential distribution with parameter θ. Let the prior distribution of θ be improper with "pdf" $1/\theta, \theta > 0$. Use the alternative parameterization of the exponential. That is:

$$f(x;\theta) = \theta e^{-x\theta} \qquad E[X] = 1/\theta, \quad Var[X] = 1/\theta^2$$

   (a) (+10 points) Find the posterior distribution of θ and the posterior mean of θ.

   (b) (+5 points) Find the MLE of θ.

   (c) (+5 points) Why do you think the Bayesian and Frequentist answers are the same here? (You might try taking the expected value of Θ to understand why the prior is improper.)

2. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from a Poisson distribution for which the mean, λ, is unknown. We want to test the following hypotheses:

$$H_0 : \lambda \leq 1$$
$$H_1 : \lambda > 1$$

Suppose also that we have a sample of size 10.

(a) (+10 points) Find a UMP test at a reasonable level of significance. The test should be specific/well defined (that is, I want you to tell me what the "c" is.)

(b) (+5 points) Justify (in words) why the test you have found above is UMP.

(c) (+ 5 points) Give the level of significance.

(d) You collect the following data:

i	1	2	3	4	5	6	7	8	9	10
X_i	2	3	2	2	0	1	0	3	1	1

(+5 points) Make a conclusion about whether or not to reject the null hypothesis. Note that regardless of your answer for (a), you will get full credit for this problem as long as your answer is reasonable.

3. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with pmf $P(X = x; \theta) = \theta(1-\theta)^x, x = 0, 1, 2, \ldots,$ zero elsewhere, where $0 \leq \theta \leq 1$. (Notice $X \sim$ NegBin(1,θ).)

   (a) (+5 points) Find a complete sufficient statistic for θ. Justify, in words, why it is complete and sufficient for θ.

   (b) (+5 points) Show that the function:

   $$Y_2 = h(X_1) = \begin{cases} 1 & \text{if } X_1 = 0 \\ 0 & \text{else} \end{cases}$$

   is unbiased for θ.

   (c) (+10 points) Find the MVUE of θ.

   (d) (+5 points) Justify (in words) that the estimator you found in part (c) is the MVUE. Note, you should be able to answer this question completely without actually doing part (c).

4. Suppose we have a single observation, X, from a Beta(θ,1). We would like to test:

$$H_0 : \theta = 2$$
$$H_1 : \theta = 3$$

(a) (+5 points) Find the test that minimizes $2\alpha + 3\beta$.

(b) (+5 points) Find $2\alpha + 3\beta$.

(c) (+5 points) Why is it unlikely that we would actually use the test from part (a)?