

Name: \_\_\_\_\_

We know that if  $Z \sim N(0,1)$ , then  $Y = |Z|$  has the pdf:

$$f_Y(y) = \frac{2}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} \quad y > 0$$

Find the pdf of  $X = Y^2$ . If you can, identify the distribution of  $X$  by name. (Hint: use the idea of finding the pdf of  $X$  from the pdf of  $Y$ : transformations. You'll also need to know  $dy/dx$ .)

**Solution**

$$\begin{aligned} X &= Y^2 = g(Y) \\ Y &= \sqrt{X} = g^{-1}(X) \\ f_X(x) &= f_Y(g^{-1}(x)) |g^{-1}'(x)| = f_Y(g^{-1}(x)) |dy/dx| \\ &= \frac{2}{\sqrt{2\pi}} e^{-\frac{1}{2}x} \frac{1}{2} x^{-1/2} \\ &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x} x^{-1/2} \quad x > 0 \end{aligned}$$

$$X \sim \chi_1^2$$