Professor Hardin
Math 152 – Fall 2006
warm-up # 10

Name: ______________________

Let's say we're watching buses go by, and we're trying to decide if there are at least 4 per hour.

$$X_i \sim \text{Poisson}(\theta)$$
$$H_o : \theta \leq 4$$
$$H_1 : \theta > 4$$

Find the LRT for this test.
Solution: Note, MLE of $\theta = \hat{\theta} = \overline{X}$

$$\begin{aligned}
f(x_i|\theta) &= \frac{\theta^{x_i} e^{-\theta}}{x_i!} \\
f(\underline{x}|\theta) &= \frac{\theta^{\sum x_i} e^{-n\theta}}{\prod x_i!} \\
L(\hat{\theta}) &= \frac{\overline{x}^{\sum x_i} e^{-n\overline{x}}}{\prod x_i!} \\
L(\theta_o) &= \frac{4^{\sum x_i} e^{-n4}}{\prod x_i!} \\
\Lambda &= \frac{4^{\sum x_i} e^{-n4}}{\overline{x}^{\sum x_i} e^{-n\overline{x}}} \\
&= \left(\frac{4e}{\overline{x}}\right)^{\sum x_i} e^{-n4}
\end{aligned}$$

Our critical region: C $= \{(X_1, X_2, \ldots, X_n) | \left(\frac{4e}{\overline{x}}\right)^{\sum x_i} \leq k\}$ is a LRT.

If we find k such that $P(\underline{X} \in C | H_o \text{ true}) = 0.05$, then the critical region is a LRT of size α. How do we find k?

Let $g(a) = \left(\frac{4e}{a}\right)^{na} \Rightarrow g'(a) < 0 \Rightarrow$ as $a \to \infty$, $\left(\frac{4e}{a}\right)^{na}$ decreases.

So, our critical region becomes: C $= \{(X_1, X_2, \ldots, X_n) | \sum X_i \geq k'\}$ is a LRT.

To find k' we use $\sum X_i \sim$ Poisson or the CLT.