Professor Hardin
Math 152 – Fall 2006
warm-up # 11

Name: ________________________

Let $X_1, X_2, \ldots, X_n$ be a random sample fromt he pdf:

$$f(x;\theta) = e^{-(x-\theta)} \qquad \theta < x$$

Using definition 7.2.1 (not theorem 7.2.1) show that the minimum is a sufficient statistic for θ.

Solution: let $Y_1 = \min(X_i)$.

Note:

$$\begin{aligned} F_X(x) &= \int_\theta^x e^{-(w-\theta)} dw \\ &= -e^{-(w-\theta)}|_\theta^x \\ &= 1 - e^{-(x-\theta)} \end{aligned}$$

$$\begin{aligned} f(x_i;\theta) &= e^{-(x-\theta)} I_{[\theta,\infty]}(x_i) \\ f(\underline{x};\theta) &= e^{-\sum_i x_i + n\theta} I_{[\theta,\infty]}(\min x_i) \end{aligned}$$

$$\begin{aligned} f_{Y_1}(y_1;\theta) &= n[1-(1-e^{-(y_1-\theta)})]^{n-1} e^{-(y_1-\theta)} I_{[\theta,\infty]}(y_1) \\ &= n[e^{-(y_1-\theta)}]^n I_{[\theta,\infty]}(y_1) \\ &= n e^{-ny_1+n\theta} I_{[\theta,\infty]}(y_1) \end{aligned}$$

$$\frac{f(\underline{x};\theta)}{f_{Y_1}(y_1;\theta)} = \frac{e^{-\sum_i x_i}}{ne^{-ny_1}}$$

Because the preceeding ratio does not depend on θ, we know $Y_1 = \min(X_i)$ is sufficient for θ.