Professor Hardin
Math 152 – Fall 2006
warm-up # 12

Name: ______________________

Let $X_1, X_2, \ldots, X_n$ be a random sample from the Uniform$[0, \theta]$ pdf:

$$f(x;\theta) = \frac{1}{\theta} \qquad x < \theta$$

Notice that the distribution of $Y = \max(X_i)$ is:

$$f(y;\theta) = \frac{ny^{(n-1)}}{\theta^n}$$

1. Find the sufficient statistic for θ.
2. Find the MLE of θ.
3. Find an unbiased estimator of θ based on the sufficient statistic.

Solution:

1.

$$\begin{aligned} f(\underline{x};\theta) &= \left(\frac{1}{\theta}\right)^n \qquad \text{all } x_i < \theta \\ &= \left(\frac{1}{\theta}\right)^n I_{[0,\theta]}(\max x_i) \end{aligned}$$

From theorem 7.2.1, $Y = \max(x_i)$ is sufficient for θ.

2. Using a plot of the likelihood function, we can see that the likelihood is maximized when theta is as small as possible without being smaller than any of the x_i. So, the MLE is $\hat{\theta} = \max(x_i)$.

3.

$$\begin{aligned} E[Y] &= \int_0^\theta y \frac{ny^{(n-1)}}{\theta^n} dy \\ &= \frac{n}{n+1} \frac{y^{n+1}}{\theta^n} |_0^\theta \\ &= \frac{n}{n+1}\theta \end{aligned}$$

We notice that Y is not an unbiased estimator of θ. But $Y_2 = \frac{n+1}{n}Y$ **is** an unbiased estimator of θ!