

Name: \_\_\_\_\_

Suppose that a single observation is taken from the following pdf:

$$f(x|\theta) = \begin{cases} 2(1-\theta)x + \theta & 0 \leq x \leq 1, \quad 0 \leq \theta \leq 2 \\ 0 & \text{else} \end{cases}$$

The following hypotheses are to be tested:

$$H_o : \theta \geq 1 \\ H_1 : \theta < 1$$

Let  $\delta : \{\text{reject } H_o \text{ if } X \geq 0.9\}$ .

1. Determine the power function of the test ( $\pi(\theta|\delta)$ .)
2. Determine the size of the test. (Note: size =  $\alpha(\delta) = \sup_{\theta \in \Omega_0} \pi(\theta|\delta)$  . )

### Solution

1.

$$\begin{aligned} \pi(\theta|\delta) &= P(\text{rejecting } H_o \mid \theta) \\ &= P(X \geq 0.9 \mid \theta) \\ &= \int_{0.9}^1 2(1-\theta)x + \theta \, dx \\ &= 2(1-\theta) \frac{1}{2} x^2 \Big|_{0.9}^1 + \theta x \Big|_{0.9}^1 \\ &= (1-\theta) - (1-\theta)(0.9)^2 + \theta(1-0.9) \\ &= (1-\theta)(1-0.81) + 0.1\theta \\ &= 0.19 - 0.19\theta + 0.1\theta \\ \pi(\theta|\delta) &= 0.19 - 0.09\theta \end{aligned}$$

2.

$$\begin{aligned} \text{size} = \alpha(\delta) &= \sup_{\theta \in \Omega_0} \pi(\theta|\delta) \\ &= \pi(\theta = 1|\delta) \\ &= 0.19 - 0.09 = 0.10 \end{aligned}$$