Math 152 – Fall 2007
Professor Hardin
warm-up # 13

Name: ________________________

Assume weights of cereal in a 10oz box are normally distributed. Test whether or not the box is accurate (no preconceived notion of directionality) at a size of 0.05. Also, approximate the p-value associated with the test. (Data: $n = 16, \overline{x} = 10.4$oz, $s = 0.85$oz.)

Solution

$$
\begin{aligned}
H_0 : \mu &= 10\text{oz} \\
H_1 : \mu &\neq 10\text{oz} \\
\delta &: \left\{ \text{reject } H_0 \text{ if } T \in C \text{ where } T = \frac{|\overline{X} - 10|}{s/\sqrt{n}} \right\} \\
C &= \left\{ \frac{|\overline{X} - 10|}{s/\sqrt{n}} > c \right\} \\
& \quad \text{that is, reject } H_0 \text{ if } |\overline{X} - \mu_0| \text{ is really big} \\
\alpha(\delta) &= \sup_{\mu \in \Omega_0} \pi(\mu|\delta) \\
&= P\left(\frac{|\overline{X} - 10|}{s/\sqrt{n}} > c | \mu = 10 \right) \\
&= P\left(-c < \frac{\overline{X} - 10}{s/\sqrt{n}} \right) \\
&= P(-c < t_{15} < c) = 0.05 \\
c &= 2.131
\end{aligned}
$$

That is, reject H_0 if $\overline{X}$ not $\in 10 \pm 0.453$. We cannot reject the null hypothesis at $\alpha = 0.05$. At the 0.05 level, we do not have evidence to say that cereal boxes weigh, on average, something different from 10oz.

$$
\begin{aligned}
\text{p-value} &= P(T > \hat{T}) \\
&= P\left(\frac{|\overline{X} - 10|}{s/\sqrt{n}} > \frac{10.4 - 10|}{.85/\sqrt{16}} \right) \\
&= 2P(t_{15} > 1.88) \\
1.752 \leq \quad & 1.88 \quad \leq 2.131 \\
2 * 0.025 \leq \quad & \text{p-value} \quad \leq 2 * 0.05 \\
0.05 \leq \quad & \text{p-value} \quad \leq 0.1
\end{aligned}
$$

We have moderate evidence (not strong) to say that cereal boxes are different from 10oz, on average.