

Name: \_\_\_\_\_

Suppose that a single observation is taken from a Bernoulli pdf. The following hypotheses are to be tested:

$$H_o : p = 0.3$$

$$H_1 : p = 0.6$$

Find the test  $\delta$  that minimizes the sum of the type I error and type II error (i.e., the quantity  $\alpha(\delta) + \beta(\delta)$ .)

**Solution**

$$\begin{aligned} \delta & : \{ \text{reject } H_0 \text{ if } af_0(X) < bf_1(X) \} \\ af_0(X) & = 1p^X(1-p)^{1-X} \\ & = 0.3^X 0.7^{1-X} \\ bf_1(X) & = 1p^X(1-p)^{1-X} \\ & = 0.6^X 0.4^{1-X} \\ \frac{af_0(X)}{bf_1(X)} & < 1 \\ \left(\frac{0.3}{0.6}\right)^X \left(\frac{0.7}{0.4}\right)^{1-X} & < 1 \\ 0.5^X 1.75^{1-X} & < 1 \\ X \ln(.5) + (1-X) \ln(1.75) & < 0 \\ X(-\ln(2) - \ln(1.75)) & < -\ln(1.75) \\ X & > \frac{\ln(1.75)}{\ln(2) + \ln(1.75)} = 0.447 \\ \delta & : \{ \text{reject } H_0 \text{ if } X > 0.447 \} \end{aligned}$$