Professor Hardin
Math 152 – Fall 2006
warm-up # 15

Name: ________________________

Suppose that a random sample, $X_1, X_2, \ldots, X_n$, is taken from a Gamma(α, β) distribution where $\alpha > 0$ is known and $\beta > 0$ is unknown. Using the Neyman-Pearson theorem & definition 8.2.1, find a UMP test for:

$$H_0 : \beta \leq 47$$
$$H_1 : \beta > 47$$

Solution: Let $\beta^* > 47$

$$\begin{aligned} f(\mathbf{x}|\alpha,\beta) &= \frac{1}{\Gamma(\alpha)^n \beta^{n\alpha}} \prod x_i^{\alpha-1} e^{-\sum x_i/\beta} \\ \frac{L(47;\mathbf{x})}{L(\beta^*;\mathbf{x})} &= \left(\frac{\beta^*}{47}\right)^{n\alpha} e^{-\sum x_i(1/47 - 1/\beta^*)} \\ &\leq k \quad \text{iff } \sum x_i \geq c_1 \end{aligned}$$

Note that the relationship holds for any $\beta^* > 47$. So, as long as we can find c_1, we know (by definition 8.2.1) that the critical region:

$$C = (\mathbf{X}| \sum X_i \geq c_1)$$

is the uniformly most powerful critical region.

Let $\alpha = 0.01$. We know $\sum X_i \sim$ Gamma$(n\alpha, \beta)$. So, let $Y = \sum X_i$. Find c_1 such that:

$$0.01 = P(Y > c_1)$$

where $Y \sim$ Gamma$(n\alpha, \beta)$.