

Name: _____

Suppose that $X_1, X_2, \dots, X_n \sim f(x|\theta)$. Where θ is unknown. The joint pdf, $f(\underline{x}|\theta)$ is MLR in $T(\underline{X})$. Let θ_0 be a specified value of θ , and suppose that the following hypotheses are to be tested:

$$H_0 : \theta \geq \theta_0$$

$$H_1 : \theta < \theta_0$$

Let c be a constant such that $P(T \leq c | \theta = \theta_0) = \alpha_0$. Show that the test procedure which rejects H_0 if $T \leq c$ is UMP test at level α_0 .

(Hint: let $\gamma = -\theta$.)

Solution

Letting $\gamma = -\theta$:

$$H_0 : \gamma \leq -\theta_0$$

$$H_1 : \gamma > -\theta_0$$

We know that $f(\underline{x}|\gamma)$ will be decreasing in T ; so $f(\underline{x}|\gamma)$ will be increasing in $-T$. Therefore, the test that rejects H_0 when $-T \geq c'$ is UMP for the above hypotheses. Multiplying by negative one, the test that rejects H_0 when $T \leq c$ is UMP. If c is chosen to satisfy the equation in the problem, it follows that the level of the test is α_0 .