Professor Hardin
Math 152 – Fall 2006
warm-up # 17

Name: ________________________

Let θ denote the average number of defects per 100 feet of a certain type of magnetic tape. Suppose that the value of θ is unknown, and the prior distribution of θ is a gamma distribution with parameters $\alpha = 2$ and $\beta = 10$. When a 1200-foot roll of this tape is inspected, exactly four defects are found. Determine the posterior distribution of θ.

Use the alternative gamma parameterization:

$$f(x|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-x\beta}$$

Solution:
Note that we've taken 12 observations and seen $\sum x_i = 4$.

$$\begin{aligned}
f(\underline{x}|\theta) &= \theta^{\sum x_i} e^{-n\theta} / \prod x_i \\
h(\theta) &= \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\theta\beta} \\
k(\theta|\underline{x}) &\propto \theta^{\sum x_i + \alpha - 1} e^{-\theta(n+\beta)} \\
\theta|\underline{x} &\sim \text{Gamma}(\textstyle\sum x_i + \alpha, n + \beta) \\
&\sim \text{Gamma}(6, 22)
\end{aligned}$$