Math 152 – Fall 2007
Professor Hardin
warm-up # 17

Name: ________________________

Let's say we have a population of radon detectors whose accuracy we'd like to test. Because we don't want to open all the packages, we randomly select 12 and put them in a room with 105 picocuries per liter (pCi/l) of radon. We are worried that the detectors will underestimate the amount of radon in the room. Our data give us: $\overline{x} = 104.13$ pCi/l and $s = 9.20$ pCi/l. Test the appropriate hypotheses.

Note: is $\overline{X}$ still MLR for the pdf? Is that relevant?

Solution
$H_0 : \mu \geq 105$ pCi/l
$H_1 : \mu < 105$ pCi/l

Reject if $\overline{X}$ small or if $U = \frac{\overline{X}-\mu_o}{s/\sqrt{n}}$ is small.

$$\begin{aligned} \delta &: \{\text{Reject } H_0 \text{ if } U \leq c\} \\ U &\sim t_{11} \text{ when } H_0 \text{ is true} \\ \delta &: \{\text{Reject } H_0 \text{ if } U \leq -1.796 \text{is a level } \alpha_0 \text{ test}\} \end{aligned}$$

Our sample U is -0.327, so we do not reject H_0.

For a fixed σ, $\overline{X}$ will still be MLR for the pdf. However, when we have two parameters to estimate we should think about how the likelihood ratio changes as both parameters change. Because the relationship between the statistic and the likelihood ratio is not obvious (when the likelihood ratio is a function of two parameters), we do not use theorem 8.3.1 to find a UMP test based on the MLR statistic.