

Name: _____

Suppose that we want to estimate the true proportion of freethrows Shaquille O’Neal can make. Not being big basketball fans, we don’t really know his ability. So, we’ll use the prior information that his true ability of making a freethrow is uniformly distributed from zero to one.

We get to take a random sample of size n freethrows (or really, he takes the freethrows). Assume the n freethrows have a binomial distribution where the true probability of making the basket is the unknown quantity in which we’re interested. Find the posterior distribution of that quantity. (Note that it is equally valid to think about the n throws as Binomial or n separate Bernoulli measurements.)

Solution:

$$X_1, X_2, X_3 \sim \text{Bernoulli}(\theta)$$

$$f(\underline{x}|\theta) = \theta^y(1 - \theta)^{(n-y)} \quad y = \sum_i x_i$$

$$\xi(\theta) = 1 \quad 0 \leq \theta \leq 1$$

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$$\theta|\underline{x} \sim \text{Beta}(\alpha = y + 1, \beta = n - y + 1)$$