Math 152 – Fall 2006
Professor Hardin
warm-up # 2

Name: _____

Let $Y_1 < Y_2 < \ldots < Y_5$ be the order statistics associated with 5 (n) independent observations $X_1, X_2, \ldots, X_5$, each with pdf

$$f_X(x) = 2x \qquad 0 < x < 1$$

Find the cdf and pdf of the max of the Xs, that is, of Y_5.

Soultion

$$\begin{aligned} F_{Y_5}(y) &= P(Y_5 \leq y) = P(\text{all } X_i < y) = \prod_i P(X_i < y) = (F_X(y))^5 = (\int_0^y 2x dx)^5 = (y^2)^5 \\ &= y^{10} \qquad 0 < y < 1 \\ f_{Y_5}(y) &= 10y^9 \qquad 0 < y < 1 \end{aligned}$$