Professor Hardin
Math 152 – Fall 2007
warm-up # 21

Name: _____

Consider the situation given in the handout, "Patterns in DNA". We would like to test whether the data conform to a Poisson process. That is, we want to know if, for a given rate (# of palindromes / segment), the count in each segment is distributed according to a Poisson distribution.

1. Because we don't know the rate, we have to estimate it. We use the MLE for λ. Using the data on page 84, find $\hat{\lambda}$.
2. Using the MLE from part (a), calculate the expected number of intervals that would have exactly 6 palindromes.
3. The test statistic and p-value are given to you on page 85. Report these values and give a conclusion in words.

Solution:

1. The MLE is just $\overline{X} = 5.16$. This should seem about right given the data on page 85.
2. $$\begin{aligned} P(\text{6 palindromes in an interval of length 4000}) &= \lambda^6 e^{-\lambda}/6! \\ &= (5.16)^6 e^{-5.16}/6! = 0.1505 \end{aligned}$$

   So, the expected number of intervals with 6 palindromes is $57 * 0.1505 = 8.58$.
3. $\chi^{2*} = 1.0$, p-value = 0.98. Given the small test statistic and large p-value, we have no evidence against the null hypothesis. So, we do not reject H_0, and say that the Poisson model is reasonable for this data. (Note that I don't say that we are certain that the Poisson model is the truth.)