Math 152 – Fall 2007
Professor Hardin
warm-up # 3

Name: ________________________________

Continuing the example from class with θ denoting the average number of defects per 100 feet of tape, the prior on θ is a gamma distribution with $\alpha = 2, \beta = 10, E[\theta] = 0.2$. We found that the likelihood of the data was Poisson(θ). Also, we found that the Bayes estimate of θ is the posterior mean: $\hat{\theta}_b = E[\theta|X] = \frac{\sum X_i + 2}{n+10}$.

Note that the frequentist estimate of θ is $\hat{\theta}_f = \frac{\sum X_i}{n}$.

$$\begin{aligned}\text{frequentist MSE}(\hat{\theta}_f) &= \text{Var}(\hat{\theta}_f) + \text{bias}^2(\hat{\theta}_f)\\ \text{here, bias}(\hat{\theta}_f) &= 0 \quad \text{(prove at home)}\end{aligned}$$

$$\begin{aligned}\text{Bayesian MSE}(\hat{\theta}_b) &= \text{Var}(\theta|X) + \text{bias}^2(\theta|X)\\ \text{here, bias}(\theta|X) &= 0 \quad \text{(prove at home)}\end{aligned}$$

1. Find MSE($\hat{\theta}_f$) in the frequentist sense
2. Find MSE($\hat{\theta}_b$) in the Bayesian sense
3. If we want to compare $\hat{\theta}_f$ and $\hat{\theta}_b$, why do we need to compute MSE($\hat{\theta}_b$) in the ***frequentist*** sense?

Solution:

1.

$$\begin{aligned}MSE(\hat{\theta}_f) &= \text{Var}(\hat{\theta}_f) = \text{Var}(\overline{X})\\ &= \frac{\theta}{n}\end{aligned}$$

2.

$$\begin{aligned}MSE(\hat{\theta}_b) &= \text{Var}(\theta|X)\\ &= \frac{\sum X_i + 2}{(n+10)^2} \quad \text{(i.e., the posterior variance)}\\ &= \frac{8}{22^2}\end{aligned}$$

3. The MSE in the frequentist sense is a function of θ; the MSE in the Bayesian sense is a function of X. If we want to compare apples to apples, we must think of $\hat{\theta}_b$ as a frequentist estimator, and compute the frequentist MSE. It's impossible to think of $\hat{\theta}_f$ as a Bayesian estimator, because we can't just impose a prior (whereas in the other direction, it's easy to "forget" about the prior).