Math 152 – Fall 2006
Professor Hardin
warm-up # 5

Name: _____

1. Why do we do maximum likelihood estimation?
2. How do we do maximum likelihood estimation (in words)?
3. Let $X_1, X_2, \ldots, X_n$ be a random sample from the following pdf:

$$f(x;\theta) = \theta x^{\theta-1} \quad 0 < x < 1, \quad 0 < \theta < \infty$$

Find the mle, $\hat{\theta}$, for θ.

Solution

1. The goal of maximum likelihood estimation is to find functions of our data that are good (using some measure of good) estimates of the parameter (or function of the parameter) of interest. Many times (without MLE or MOM), it isn't obvious how we should use our data to find the appropriate function of the data to estimate the parameter. Also, MLEs have lovely properties like: (a) a function of an MLE is an MLE for that same function of the parameter and (b) MLEs are consistent estimators of the parameter.
2. To find the MLE we simply maximize the likelihood function with respect to the parameter of interest. Really, we're asking the question "what parameter values would have been most likely to produce the data we saw?" Remember, this is opposite of the question we typically ask in probability, "what data is most likely to come from our specified probability model?"
3.

$$\begin{aligned}
L(\theta) &= \prod_{i=1}^{n} \theta x_i^{\theta-1} \\
l(\theta) &= \sum_{i=1}^{n} [\ln(\theta) + (\theta-1)\ln(x_i)] \\
&= n\ln(\theta) + (\theta-1)\sum_{i=1}^{n} \ln(x_i) \\
\frac{\partial l(\theta)}{\partial \theta} &= \frac{n}{\theta} + \sum_{i=1}^{n} \ln(x_i) \\
\hat{\theta} &= \frac{-n}{\sum_{i=1}^{n} \ln(x_i)}
\end{aligned}$$

Notice that the estimate **looks** negative, but it is actually positive! Why?