Math 152 – Fall 2006
Professor Hardin
warm-up # 6

Name: ________________________

Let $X_1, X_2, \ldots, X_n$ be iid from the pdf (note, it's a Poisson distribution):

$$f(x;\lambda) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad \lambda > 0$$

1. Find the Fisher information, $I(\lambda)$.
2. Is $\overline{X}$ an efficient estimator? Show.

Solution

1.

$$\begin{aligned}
\frac{\partial \ln f(x;\lambda)}{\partial \lambda} &= \frac{\partial}{\partial \lambda}(x \ln \lambda - \lambda - \ln x!) \\
&= \frac{x}{\lambda} - 1 = \frac{x-\lambda}{\lambda} \\
I(\lambda) &= Var\Big(\frac{\partial \ln f(X;\lambda)}{\partial \lambda}\Big) \\
&= \frac{1}{\lambda^2} Var(X) \\
&= \frac{1}{\lambda}
\end{aligned}$$

2.

$$\begin{aligned}
E(\overline{X}) &= \lambda \\
Var(\overline{X}) &= \frac{1}{n} Var(X) = \frac{\lambda}{n}
\end{aligned}$$

$\overline{X}$ is an unbiased estimator for λ. We know that the RCLB for an unbiased estimator is $\frac{1}{nI(\lambda)}$. Because the variance of $\overline{X}$ attains the RCLB, $\overline{X}$ is an efficient estimator.