Math 152 – Fall 2007
Professor Hardin
warm-up # 6

Name: ________________________________

Let X_1 and X_2 be independent random variables. Let X_1 and $Y = X_1 + X_2$ have χ^2 distributions with r_1 and r degrees of freedom, respectively. Here $r_1 < r$. Show that X_2 has a $\chi^2_{r_2}$ distribution where $r_2 = r - r_1$. Hint: use moment generating functions.

Solution

$$\begin{aligned}
M_{X_1}(t) &= E(e^{tX_1}) = (1-2t)^{-r_1/2} \\
M_Y(t) &= E(e^{tY}) = (1-2t)^{-r/2} \\
M_{X_2}(t) &= E(e^{tX_2}) \\
&= \frac{E(e^{tX_2}) * E(e^{tX_1})}{E(e^{tX_1})} \\
&= \frac{E(e^{t(X_2+X_1)})}{E(e^{tX_1})} \qquad \text{because } X_1 \text{ and } X_2 \text{ are independent} \\
&= \frac{E(e^{tY})}{E(e^{tX_1})} \\
&= \frac{M_Y(t)}{M_{X_1}(t)} \\
&= (1-2t)^{-r_2/2} \\
\rightarrow X_2 &\sim \chi^2_{r_2}
\end{aligned}$$