Math 152 – Fall 2006
Professor Hardin
warm-up # 7

Name: ________________________

The Johns Hopkins Regional Talent Searches give the SAT to 13-year-olds. Let's say that 12 boys and 15 girls took the test. The data are as follows:

Group	$\overline{x}$	s
Boys	416	87
Girls	386	74

Assuming that SAT scores are normally distributed, find a 90% confidence interval for the ratio of standard deviations for SAT scores taken by boys vs. girls. INTERPERET the interval.

Note: (from page 185), an F distribution is created by using the ratios of 2 independent Chi-square random variables divided by their degrees of freedoms. We all remember that if we have normal data, $(n-1)S^2/\sigma^2 \approx \chi^2_{(n-1)}$. So,

$$F = \frac{\frac{(n-1)S_1^2}{\sigma_1^2}/(n-1)}{\frac{(m-1)S_2^2}{\sigma_2^2}/(m-1)} = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}$$

has an F distribution with df={(n-1),(m-1)}.

Solution:
$f_{0.95,14,12} = 2.64$
$f_{0.95,12,14} = 2.53$

$$P(a \leq F_{14,12} \leq b) = .9 \rightarrow b = 2.64$$
$$P(1/a \geq 1/F_{14,12} \geq 1/b) = .9 \rightarrow P(1/b \leq F_{12,14} \leq 1/a) = .9 \rightarrow a = 1/2.53 = 0.395$$

We want:

$$\frac{\sigma_b^2}{\sigma_g^2} \in \left(\frac{s_b^2}{s_g^2} \frac{1}{f_{.95,12,14}}, \frac{s_b^2}{s_g^2} f_{.95,14,12} \right) \rightarrow \left(\frac{87^2}{74^2} \frac{1}{2.53}, \frac{87^2}{74^2} \cdot 2.64 \right) \rightarrow (0.546, 3.650)$$

$$\frac{\sigma_b}{\sigma_g} \in (\sqrt{0.546}, \sqrt{3.65}) \rightarrow (0.739, 1.91)$$

We are 90% confident that the true ratio of SAT standard deviations for boys vs. girls is between 0.739 and 1.91. (No units.)

Note that the F-interval is much more sensitive (less robust) to the normality assumptions than the corresponding t-intervals for μ. For this reason, resarchers are hesitant to put very much weight on the F-intervals.