Math 152 – Fall 2007
Professor Hardin
warm-up # 7

Name: ________________________________

According to many investors, foreign stocks have the potential for high yield, but the variability in their dividends may be greater than what is typical for American companies. If **we believe foreign stock prices are distributed similarly** to American stock prices:

1. How likely is it that a sample of 10 foreign stocks would produce a standard deviation that is 50% bigger than American stocks?
2. What is the value k such that the sample mean of 10 foreign stocks is no more than k standard deviations above the true mean μ with probability 0.90?
   p.s. How would this problem have been different if we had know the true variance σ^2?

Solution

1.

$$
\begin{aligned}
P\left(\frac{\hat{\sigma}}{\sigma} > 1.5\right) &= \ ? \\
\frac{\sum(X_i - \overline{X})^2}{\sigma^2} &\sim \chi^2_{n-1} \qquad \text{notice that I've assumed normality!} \\
\frac{\sum(X_i - \overline{X})^2}{\sigma^2} &= \frac{n\sum(X_i - \overline{X})^2/n}{\sigma^2} = \frac{n\hat{\sigma}^2}{\sigma^2} \\
P\left(\frac{\hat{\sigma}}{\sigma} > 1.5\right) &= P\left(\frac{\hat{\sigma}^2}{\sigma^2} > (1.5)^2\right) = P\left(\frac{n\hat{\sigma}^2}{\sigma^2} > n(1.5)^2\right) \\
&= 1 - \chi^2_{n-1}(n(1.5)^2) = 1 - \chi^2_9(22.5) \\
&< 0.01 \qquad \text{see table on page 774-775}
\end{aligned}
$$

You could have also used the MLE for σ^2, and your work would have $n-1$ instead of n (the answer would be approximately 0.0164).

2.

$$
\begin{aligned}
P(\overline{X} < \mu + ks) &= 0.9 \\
P(\frac{\overline{X} - \mu}{s} < k) &= 0.9 \\
P(\frac{\overline{X} - \mu}{s/\sqrt{n}} < k\sqrt{n}) &= 0.9 \\
\frac{\overline{X} - \mu}{s/\sqrt{n}} &\sim t_9 \\
\sqrt{n}k &= 1.383 \qquad \text{see table on page 776-777} \\
k &= 0.461
\end{aligned}
$$

If we had known σ^2 we would have been able to use a normal table instead of a t table.