Math 152 – Fall 2006
Professor Hardin
warm-up # 8

Name: _____

You are playing a coin game with a friend. You know that you're playing with one of two coins, either the fair coin or the coin that lands heads 70% of the time. You would like to know which coin you have, so you flip the coin 20 times and count the number of heads. You will reject the hypothesis that it's a fair coin if you see more than 15 heads.

1. What are the null and alternative hypotheses for this test? (Remember to define the parameter value of interest.)
2. What is the size of the test?
3. What is the power of the test?

Solution:

1.

$$\begin{aligned} & X \sim \text{Bin}(\theta, 20) \quad \theta = \text{P(heads)} \\ & H_o : \theta = 0.5 \\ & H_1 : \theta = 0.7 \end{aligned}$$

2.

$$\begin{aligned} \alpha &= \max_{\theta \in \omega_o} P_\theta[(X_1, X_2, \ldots X_n) \in C] \\ &= \max_{\theta \in \omega_o} P_\theta[X > 15] \\ &= P[X > 15 | \theta = 0.5] \\ &= P\Big[\frac{X - 10}{\sqrt{20 * .5 * .5}} > \frac{15 - 10}{\sqrt{20 * .5 * .5}}\Big] \\ &= P[Z > 2.236] = 0.013 \end{aligned}$$

3.

$$\begin{aligned} \text{power}(\theta) &= P_{\theta \in \omega_1}[(X_1, X_2, \ldots, X_n) \in C] \\ &= P_{\theta = .7}[X > 15] \\ &= P\Big[\frac{X - 14}{\sqrt{20 * .7 * .3}} > \frac{15 - 14}{\sqrt{20 * .7 * .3}}\Big] \\ &= P[Z > 0.488] = 0.313 \end{aligned}$$