Math 152 – Fall 2007
Professor Hardin
warm-up # 8

Name: ________________________________

Let $V \sim \chi^2_\nu$, $W \sim \chi^2_\omega$, an F statistic (not a symmetric distribution) is defined as:

$$U = \frac{V/\nu}{W/\omega}$$
$$U \sim F_{\nu,\omega}$$

We have two samples: $X_1, X_2, \ldots, X_n \sim N(\mu_1, \sigma_1^2)$; $Y_1, Y_2, \ldots, Y_m \sim N(\mu_2, \sigma_2^2)$. Find a 90% confidence interval for σ_2^2/σ_1^2. (How would you find a 90% confidence interval for σ_2/σ_1? What about for σ_1/σ_2?)

Solution

$$P(c_1 \leq F_{(n-1),(m-1)} \leq c_2) = 0.9$$

$$P\left(c_1 \leq \frac{\sum(X_i-\overline{X})^2/\sigma_1^2(n-1)}{\sum(Y_j-\overline{Y})^2/\sigma_2^2(m-1)} \leq c_2\right) = 0.9$$

$$P\left(c_1((n-1)/(m-1))\frac{\sum(Y_j-\overline{Y})^2}{\sum(X_i-\overline{X})^2} \leq \frac{1/\sigma_1^2}{1/\sigma_2^2} \leq c_2((n-1)/(m-1))\frac{\sum(Y_j-\overline{Y})^2}{\sum(X_i-\overline{X})^2}\right) = 0.9$$

$$P\left(c_1((n-1)/(m-1))\frac{\sum(Y_j-\overline{Y})^2}{\sum(X_i-\overline{X})^2} \leq \frac{\sigma_2^2}{\sigma_1^2} \leq c_2((n-1)/(m-1))\frac{\sum(Y_j-\overline{Y})^2}{\sum(X_i-\overline{X})^2}\right) = 0.9$$

90% CI for σ_2^2/σ_1^2: $\left(c_1((n-1)/(m-1))\frac{\sum(Y_j-\overline{Y})^2}{\sum(X_i-\overline{X})^2}, c_2((n-1)/(m-1))\frac{\sum(Y_j-\overline{Y})^2}{\sum(X_i-\overline{X})^2}\right)$

90% CI for σ_2/σ_1: $\left(\sqrt{c_1((n-1)/(m-1))\frac{\sum(Y_j-\overline{Y})^2}{\sum(X_i-\overline{X})^2}}, \sqrt{c_2((n-1)/(m-1))\frac{\sum(Y_j-\overline{Y})^2}{\sum(X_i-\overline{X})^2}}\right)$

90% CI for σ_1/σ_2: $\left(\sqrt{1/(c_2)((m-1)/(n-1))\frac{\sum(X_i-\overline{X})^2}{\sum(Y_j-\overline{Y})^2}}, \sqrt{1/(c_1)((m-1)/(n-1))\frac{\sum(X_i-\overline{X})^2}{\sum(Y_j-\overline{Y})^2}}\right)$

Note, c_1 is the .05 cutoff for an $F_{(n-1),(m-1)}$, and c_2 is the .95 cutoff for an $F_{(n-1),(m-1)}$. Here, $-c_1 \neq c_2$ because (a) the F distribution is never negative, and (b) the F distribution isn't symmetric.