Professor Hardin
Math 152 – Fall 2006
warm-up # 9

Name: _____

Back to the cereal example in class... Assume the weight of a 10 oz box of cereal is Normally distributed with mean μ and variance σ^2. To test whether or not the box is accurate (i.e., whether it really contains "10 oz" of cereal), we set up the hypotheses:
$H_o : \mu = 10\text{oz}$
$H_1 : \mu \neq 10\text{oz}$

In class we found a critical region of:

$$C = \{(X_1, X_2, \ldots, X_n) \;\Big|\; |\overline{X} - 10| > 0.452\} \quad \text{when } n = 16, s = 0.85\text{oz}$$

Find the p-value & **conclude in words** if $n = 16, s = 0.85$oz, and

1. $\overline{X} = 10.8$oz
2. $\overline{X} = 10.1$oz
3. $\overline{X} = 9.7$oz

Solution:
p-value = probability of seeing my data **or more extreme** if the null hypothesis is true (here, if $\mu = 10$oz.)

1.

$$\begin{aligned}
\text{p-value} &= P(\overline{X} > 10.8 \text{ or } \overline{X} < 9.2 | \mu = 10) \\
&= 2P(\overline{X} > 10.8 | \mu = 10) \\
&= 2P\Big(\frac{\overline{X} - \mu_o}{s/\sqrt{n}} > \frac{10.8 - \mu_o}{s/\sqrt{n}}\Big) \\
&= 2P\Big(\frac{\overline{X} - 10}{0.85/\sqrt{16}} > \frac{10.8 - 10}{0.85/\sqrt{16}}\Big) \\
&= 2P(t_{(n-1)} > 3.765) \\
&= 2P(t_{15} > 3.765) \\
&= 0.00187
\end{aligned}$$

We reject the null hypothesis and claim the box is inaccurately labeled.

2.

$$\begin{aligned}
\text{p-value} &= P(\overline{X} > 10.1 \text{ or } \overline{X} < 9.9 | \mu = 10) \\
&= 2P(\overline{X} > 10.1 | \mu = 10)
\end{aligned}$$

$$
\begin{aligned}
&= 2P\Big(\frac{\overline{X}-\mu_o}{s/\sqrt{n}} > \frac{10.1-\mu_o}{s/\sqrt{n}}\Big)\\
&= 2P\Big(\frac{\overline{X}-10}{0.85/\sqrt{16}} > \frac{10.1-10}{0.85/\sqrt{16}}\Big)\\
&= 2P(t_{(n-1)} > 0.471)\\
&= 2P(t_{15} > 0.471)\\
&= 0.6447
\end{aligned}
$$

We do not have enough evidence to reject the null hypothesis. We cannot claim the box is inaccurately labeled.

3.

$$
\begin{aligned}
\text{p-value} &= P(\overline{X} < 9.7 \text{ or } \overline{X} > 10.3 | \mu = 10)\\
&= 2P(\overline{X} < 9.7 | \mu = 10)\\
&= 2P\Big(\frac{\overline{X}-\mu_o}{s/\sqrt{n}} < \frac{9.7-\mu_o}{s/\sqrt{n}}\Big)\\
&= 2P\Big(\frac{\overline{X}-10}{0.85/\sqrt{16}} < \frac{9.7-10}{0.85/\sqrt{16}}\Big)\\
&= 2P(t_{(n-1)} < -1.412)\\
&= 2P(t_{15} < -1.412)\\
&= 0.178
\end{aligned}
$$

We do not have enough evidence to reject the null hypothesis. We cannot claim the box is inaccurately labeled.