Name: ________________________________

Let's say we're trying to estimate the total number of soft drinks a particular vending machine will sell in a typical year. Find a 90% posterior interval for μ. Our prior information tells us:

$$\begin{aligned} \mu|\tau &\sim N(750, 15^2 = \frac{1}{(1/5)(1/45)}) \\ \tau &\sim \text{Gamma}(1, 45) \end{aligned}$$

$\mu_0 = 750, \lambda_0 = 1/5, \alpha_0 = 1, \beta_0 = 45.$
Our random sample of 10 weeks gives $\overline{x} = 692$ cans, $s^2 = \frac{14400}{9} = 1600$ cans^2.

Solution
Our posterior parameters are:

$$\begin{aligned} \mu_1 &= \frac{\lambda_0\mu_0 + n\overline{x}}{\lambda_0 + n} = \frac{750/5 + 6920}{1/5 + 10} = 693.14 \\ \lambda_1 &= \lambda_0 + n = 10.2 \\ \alpha_1 &= \alpha_0 + n/2 = 1 + 5 = 6 \\ \beta_1 &= \beta_0 + \sum(x_i - \overline{X})^2/2 + \frac{n\lambda_0(\overline{x} - \mu_0)^2}{2(\lambda_0 + n} \\ &= 45 + 14400/2 + \frac{10(1/5)(692 - 750)^2}{2((1/5) + 10)} = 7574.8 \end{aligned}$$

For a 90% posterior interval, we need critical values from a t_{12} distribution:

$$P(t_{12} \leq 1.782) = \mathbf{0.95}$$
$$P\Big(\mu_1 - 1.782\Big(\frac{\beta_1}{\lambda_1\alpha_1}\Big)^2 \leq \mu \leq \mu_1 + 1.782\Big(\frac{\beta_1}{\lambda_1\alpha_1}\Big)^2\Big) = 0.9$$
$$P(673.31 \leq \mu \leq 712.97) = 0.9$$

There is a 90% probability that the average number of cans sold per year is between 673.31 cans and 712.97 cans.