

3.1.1

$$H_o : p = .2$$

$$H_1 : p \neq .2$$

$$\text{p-value} = 2 \cdot P(X \leq 0 | p = .2) = 2 \cdot 0.018 = 0.036$$

It seems unlikely that we wouldn't see any with characteristic A. So, we reject H_o and say that this characteristic is probably not prevalent at 20% in this species.

3.1.2

$$H_o : p \leq 0.1$$

$$H_1 : p > 0.1$$

$$\text{p-value} = P(X \geq 6 | p = .1) = 1 - 0.9967 = 0.0033$$

We can reject H_o that no more than 10% of the cars in the population are unsafe. That is, we believe that more than 10% of the cars in the population are unsafe.

In terms of the binomial assumptions, we wonder if these cars are really independent (and might not have been taken off the assembly line all together?)

3.1.7

We want to find a 95% confidence interval for the proportion of companies being taken over.

(a) (0.087, 0.491) We are 95% confident that the true proportion of companies being taken over is between 0.087 and 0.491.

(b) $\hat{p} \pm z_{.975} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \rightarrow 0.25 \pm 1.96 \sqrt{3/320} \rightarrow (0.060, 0.440)$. We are 95% confident that the true proportion of companies being taken over is between 0.06 and 0.44.

3.1.10

$$H_o : p \leq 0.7$$

$$H_1 : p > 0.7$$

$$\text{p-value} = P(X \geq 18 | p = .7) = 1 - 0.9645 = 0.0355$$

We have enough evidence to reject H_o and claim that the proportion of Texas Tech law school graduates is higher than the state average. It's hard to say that this is proof, as we would see these results 3.55% of the time even if the rate is the same as the state average.

3.2.4

Exact method, but only approximately 90%:

$$0.0577 = P(Y \leq 6) \rightarrow r = 7$$

$$0.9793 = P(Y \leq 14) \rightarrow s = 15$$

$(X^{(7)}, X^{(15)}) = (117, 142)$ is an $(0.9793 - 0.0577) = 92.16\%$ CI for the median weights.

Large sample method (using 92.16% confidence, 90% confidence is okay):

$$r^* = n \cdot p^* + z_{\alpha/2} \sqrt{np^*(1-p^*)} = 10 + z_{0.0392} \sqrt{5} = 10 - 1.7624 \sqrt{5} = 6.06$$

$$s^* = n \cdot p^* + z_{1-\alpha/2} \sqrt{np^*(1-p^*)} = 10 + z_{0.9608} \sqrt{5} = 10 + 1.7624 \sqrt{5} = 13.94$$

Note: your book suggests that you round up and use the respective quantiles. This seems like an okay method, but you might think about rounding in other ways to create intervals that are inclusive or exclusive.

$(X^{(7)}, X^{(14)}) = (117, 137)$ is a 92.16% CI for the median weights.

3.2.5

$$H_o : P(X \leq 70.3in) = p = 0.95$$

$$H_1 : P(X \leq 70.3in) = p \neq 0.95$$

$$T = \# \leq 70.3in = 90$$

$$p\text{-value} = 2 \cdot P(Y \leq 90 | p = .95) = 2 \cdot P\left(\frac{Y - np}{\sqrt{np(1-p)}} \leq \frac{90 - np}{\sqrt{np(1-p)}} | p = .95\right) = 2 \cdot P(Z \leq -2.29) = 2 \cdot 0.011 = 0.022$$

It seems like there would be more people shorter than 70.3 inches in this sample if 70.3 inches was, in fact, the 95%. So, we reject the null hypothesis and say that 70.3 is not the 95%.

3.2.6

(We use large sample because $n=100$ and $p^* = .95$, so we expect 5 successes and 95 failures.)

$$r^* = n \cdot p^* + z_{\alpha/2} \sqrt{np^*(1-p^*)} = 95 + z_{0.025} \sqrt{0.45} = 95 - 1.96 \sqrt{.45} = 93.68$$

$$s^* = n \cdot p^* + z_{1-\alpha/2} \sqrt{np^*(1-p^*)} = 95 + z_{0.975} \sqrt{0.45} = 95 + 1.96 \sqrt{.45} = 96.31$$

$(X^{(94)}, X^{(97)}) = (71.1, 72.5)$ is a 95% CI for the median height. Notice that this interval does not cover 70.3 (the hypothesized value for the 95th quantile).

3.2.7

$$H_o : P(X \leq 42min) = p \geq 0.75$$

$$H_1 : P(X \leq 42min) = p < 0.75$$

$$T = \# \leq 42min = 10$$

$$p\text{-value} = 2 \cdot P(Y \leq 10 | p = .75) = 2 \cdot P\left(\frac{Y - np}{\sqrt{np(1-p)}} \leq \frac{10 - np}{\sqrt{np(1-p)}} | p = .75\right) = 2 \cdot P(Z \leq -6.93) < 0.002$$

We reject the null hypothesis. Clearly, these new recruits are slower!

3.2.8

We'll use large sample techniques because $n=50$ and $p^*=0.5$ (50 successes, 50 failures.)

$$r^* = n \cdot p^* + z_{\alpha/2} \sqrt{np^*(1-p^*)} = 25 + z_{0.025} \sqrt{12.5} = 25 - 1.96 \sqrt{12.5} = 18.07$$

$$s^* = n \cdot p^* + z_{1-\alpha/2} \sqrt{np^*(1-p^*)} = 25 + z_{0.975} \sqrt{12.5} = 25 + 1.96 \sqrt{12.5} = 31.93$$

$(X^{(19)}, X^{(32)}) = (6.26cm, 8.03cm)$ is a 95% CI for the median depth of the projectile into the armor.

P 3.2.1

$\bar{X} = 124.75, S = 24.56$ (note they use the standard deviation that is divided by n , not $n-1$)

$$t_{0.05, 19} = -1.729$$

$$90\% \text{ CI for } \mu : 124.75 \pm 1.729 \cdot 24.56 / \sqrt{19} \rightarrow (115.01 \text{ lbs}, 134.49 \text{ lbs})$$

This interval is the shortest, but it's also the one with the least confidence. We'd expect increasing the confidence to increase the interval. It seems like none of them are extremely different. Because the data are relatively symmetric (notice the smallest value is 43.75 below the mean and the largest is 40.75 above the mean) and the sample size is relatively large, it seems like the t-CI is probably a decent approximation. However, without investigating the data more thoroughly (looking at the distribution of the values), we are probably safer with the nonparametric interval.