Math 159, Spring 2005
Professor Hardin
Sample Exam #1

1. A random sample of tenth-grade boys results in the following 16 observed weights (in lbs):

   142 134 98 119 131 103 154 122 93 137 86 119 161 144 158 92

   If sorted from largest to smallest we have:

   86 92 93 98 103 119 119 122 131 134 137 142 144 154 158 161

   (a) It has been claimed that 80% of tenth-grade boys weigh more than 100 lbs. Based on the given sample, test the validity of this claim at the significance level of 0.05.

   (b) Using both small and large sample theories, give a 95% confidence interval for the probability that a tenth-grade boy has a weight over 100 lbs. Which (small or large sample) theory do you think you would use in practice? Explain.

   (c) Test the hypothesis that the median weight of all tenth-grade boys is 110.

   (d) Give a small sample 90% confidence interval for the first quartile weight of all tenth-grade boys.

   (e) Now suppose that these 16 boys went on a diet program in an attempt to lose weight. Based on the following results, is the diet an effective means of losing weight?

ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Before	142	134	98	119	131	103	154	122	93	137	86	119	161	144	158	92
After	120	130	100	120	128	98	130	123	90	130	85	116	156	144	143	96

2. In a card game, a team containing two players compete with another team of two players. Suppose that 5 games will be played. If either team wins at least 4 of the games, then the losing side will acknowledge their inferiority. Now, relate it to a hypothesis testing problem:

   (a) What are the null and alternative hypotheses?

   (b) What is the probability of making a type I error?

   (c) What is the probability of making a type II error? (Either give the error rate or explain why you would need more information.)

3. Consider the following discrete probability distribution. Find the 47^{th} quantile of X.

X	2	4	6	7	9
P(X)	0.15	?	0.25	0.20	0.10

4. Given the sign test, where T = total number of pluses and n=total number of pluses and minuses,

   (a) Argue that $t = \frac{n}{2} - \sqrt{n}$ can be used as an approximate cutoff point for our test statistic.

   (b) When does the approximation hold?

5. Assume we have an iid sample of size 20 from a continuous distribution. We want to test whether the median is greater than 47 (alternative hypothesis.)

   (a) Find the exact power for a test with $\alpha = 0.0577$ if the true parameter value is $P(X \leq 47) = 0.3$.

   (b) What is the approximate power using the normal distribution?

6. The data obtained below were collected by Cooper et al. (1967). The purpose of their investigation was to determine whether hypnotic susceptibility as measured on objective scales can be changed with practice and training. X gives the score on the scale before training, and Y gives the score after training.

subject	X	Y
1	10.5	18.5
2	19.5	24.5
3	7.5	11.0
4	4.0	2.5
5	4.5	5.5
6	2.0	3.5

(a) Test the hypothesis of no change in hypnotic susceptibility versus the alternative that hypnotic susceptibility can be increased with practice and training.

(b) Change the value in Y_3 from 11.0 to 111.0. What effect does this outlying point have on the calculations? What does this suggest about the relative insensitivity of the sign test to outliers? Can you change any one value above to produce an effect on the final decision regarding rejection of H_o? Explain.

7. The top 10 students in a large high school graduation class are ranked from 1 (best) to 10 (tenth best). Assume that each rank is equally likely to be assigned to a male or female student. Let X equal the sum of the ranks (from 1 to 10) that are assigned to female students, that is, if all the top 10 students are girls, $X = 1 + 2 + \cdots + 10 = 55$.

   (a) How many points are in the sample space.

   (b) Describe one point in the sample space.

   (c) Describe the probability function on the sample space.

   (d) Find $P(X = 0)$.

   (e) Find $P(X = 1)$.

   (f) Find f(3).

   (g) Find F(3).

8. Twelve percent of cars manufactured do not pass the inspection at the end of the assembly line and require special attention. Let X be the number of cars that do not pass inspection out of 4 cars manufactured. Assume independence from one car to the next.

   (a) Find the mean and the standard deviation of X.

   (b) Find the median and the interquartile range of X.

   Six groups of 4 cars are each manufactured, and the number of cars not passing inspection at the end of the assembly line for each group is: 0,0,0,1,1,2.

   (c) Find the sample mean and sample standard deviation.

   (d) Find the sample median and sample interquartile range.

   (e) Estimate the probability of a car not passing inspection from the data.