Course Description

Math 187 Operations Research Shahriar Shahriari

What is the content of Math 187? In linear algebra, you have solved systems of linear equations. Now consider a system of linear *inequalities*. Such a system can have many possible solutions, and we could choose the "best" solution according to some criterion. This formulation—that is optimizing a linear function subject to linear constraints—is called a *linear program* and many real life situations can be modeled using it. Scheduling problems, mixing problems, and allocation problems are a few such examples. At the heart of the class is the *simplex method* which is a way of solving linear programs. In analyzing linear programs and their solutions the idea of "duality" will be fundamental. Basically, for every linear program, there is another related linear program called its *dual*, and the interaction of the two problems will be curiously interesting. We spend about one third of the course, analyzing linear programs, the simplex algorithm, and duality. We then apply these ideas to other topics. These applications include integer programming, game theory, Markov chains, and network flows.

What is Operations Research? OR is a broad interdisciplinary field concerned with optimization (often but not always *discrete* optimization). Operations Research is one of the mathematical sciences but, generally, has its own graduate programs, its own journals, and its own professional societies. It almost could be considered a distinct discipline. Outside academia, the applications of OR are ubiquitous and, hence, many big companies even have an OR department.

Who should take Math 187? OR is *not* a core area of mathematics. You can easily get a very solid mathematics degree without taking OR, and there are no courses in the Claremont Colleges that have OR as a prerequisite. (In particular, if you are interested in going to Math grad school, then there are probably many other courses that would take priority over OR.) On the other hand, if you want to explore Operations Research (to see if you want to consider it as a career possibility), then Math 187 will give you a taste. The course could possibly be of interest to students majoring or interested in Economics and Computer Science as well. Both Linear Programming and Game Theory play an important role in Economics, and Linear Programming, Integer Programming, and Network Flows are relevant to computer scientists.

What are the prerequisites for Math 187? The only prerequisite is Linear Algebra, but this is an important prerequisite. We will be using linear algebra constantly and without a solid background in linear algebra, you will not be able to benefit from the class.

Are there overlaps with other classes? In a computer science Algorithms class, you may see linear programming, integer programming, and network flows. In Econ Micro Theory or in Game Theory for Economists you will see Game Theory. Math 186 (Stochastic OR) briefly goes over linear programming and also covers Markov Chains. The latter are also discussed in Math Modeling (Math 183) and some Linear Algebra courses. For the most part our emphasis will be a bit different. We are more concerned with developing the mathematics rigorously and precisely than we are with the applications in economics or modeling or with actual computer algorithms. There are Game theory courses in Math as well. Math 188 (Game theory at CMC, Social Choice and Decision Making at HMC) covers game theory and has a substantial overlap with this class. The "Combinatorial Optimization" course taught in the Budapest Semesters in Mathematics (BSM)

covers the same material with a slight difference in emphasis, and so you should not take OR if you have taken that course.

When should I take Operations Research? While the prerequisite for the class is only Math 60, the class does move at a brisk pace. It is probably not wise to take the course as a freshman as they are many other more core courses (e.g., vector calc, differential equations, intro to analysis, combinatorics, probability, number theory) to take first. Most students taking OR are sophomores and juniors.

What is the workload? There are two homework sets a week. Each consists of 5 substantial problems. While we do develop the material carefully, the ideas and the concepts are not overly abstract or deeply theoretical. However, figuring out the problems and mastering the subtle issues can be difficult. When past students were asked "On average, how many hours per week, did you spend on this course outside of class?", the mean of their answers was $\boxed{7}$, and the maximum answer was $\boxed{12}$. One thing to note is that the class is relentless as there are no breaks in the routine and a total of 26 assignments (and 130 homework problems) for the semester.

Will there be any real world applications? The lectures will focus on developing the theory. On the other hand, an important goal of the homework, is to train the students in the art of translating semi-real situations into mathematical models. A number of the homework problems, even though somewhat simplified, have been adapted from real examples. For a project, groups of students will present case-studies of the use of OR in the real world.

Text. The text book for my Operations Research class is

> Shahriar Shahriari, *Linear Programming, Duality, and Applications*, version 3.7, 2018.