Exam 1

October 17, 2007

Name: ________________________________

This is a closed book exam. Show all significant work and justify all your answers. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 4 problems. Relax.

1. The points $(1,0,0)$, $(0,2,0)$ and $(0,0,3)$ determine a unique plane in three dimensional Euclidean space, $\mathbb{R}^3$.
   (a) Give the equation of the plane.
   (b) Find the point on the plane which is the closest to the origin in $\mathbb{R}^3$.
   (c) Find the (shortest) distance from the plane to the origin in $\mathbb{R}^3$.
   (d) Give an expression for the line segment connecting the origin to its closest point on the plane.

2. Let D denote an open subset of the xy–plane, $\mathbb{R}^2$, and let $F\colon D \to \mathbb{R}^2$ be a vector valued function defined on D.
   (a) State precisely what it means for F to be continuous at $(x_o, y_o) \in D$.
   (b) Let f and g denote scalar fields defined on D and define $F\colon D \to \mathbb{R}^2$ by
   $$F(x,y) = \begin{pmatrix} f(x,y) \\ g(x,y) \end{pmatrix} \quad \text{for all} \quad (x,y) \in D.$$
   Prove that F is continuous at $(x_o, y_o) \in D$ if and only if f and g are both continuous at (x_o, y_o).

3. Let U denote an open subset of $\mathbb{R}^n$, and let $f\colon U \to \mathbb{R}$ be a scalar field on U.
   (a) State precisely what it means for f to be differentiable at $x \in U$.
   (b) Fix a vector v in $\mathbb{R}^n$ and define the scalar field $f\colon \mathbb{R}^n \to \mathbb{R}$ by
   $$f(x) = v \cdot x \quad \text{for all} \quad x \in \mathbb{R}^n;$$
   that is, $f(x)$ is the dot product of x with the vector v.
   Show that f is differentiable at every x in $\mathbb{R}^n$ and compute the linear map $Df(x)\colon \mathbb{R}^n \to \mathbb{R}$ for all $x \in \mathbb{R}^n$. What is the gradient of f at x for all $x \in \mathbb{R}^n$?

4. Let $r = \sqrt{x^2 + y^2}$ for all $(x,y) \in \mathbb{R}^2$. Compute ∇r and give its domain.