

## Assignment #3

Due on Friday September 21, 2007

**Read** Section 1.1 on *The Malthusian Model*, pp. 2–5, and Section 1.2 on *Nonlinear Models*, pp. 11–17, in Allman and Rhodes.

**Do** the following problems

1. Problem 1.1.11 on page 8 in Allman and Rhodes.
2. Problem 1.2.7 on page 18 in Allman and Rhodes.
3. Problem 1.2.8 on page 18 in Allman and Rhodes.
4. (*US Census Data.*) The MS Excel file CensusDataUS in the Math 36 webpage (see the courses website at <http://pages.pomona.edu/~ajr04747>) contains the total US population (in millions of people) for each year that a census has been taken in the United States.
  - (a) Use MATLAB<sup>®</sup> to get a plot of the US population as a function of  $t$ , where  $t$  is in units of 10 years since the year 1790.
  - (b) If the US population follows a Malthusian model, what would the growth rate  $\lambda$  be? Using this value of  $\lambda$ , compute the population values that the model predicts for  $t = 1, 2, 3, \dots$ . Plot the predicted and actual values on the same graph. How well do these predictions compare with the actual data?
5. (*US Census Data, continued.*) Starting with the solution to the Malthusian model:  $N_t = N_0\lambda^t$ , take logarithms on both sides to get

$$\ln N_t = \ln N_0 + t \ln(\lambda).$$

Thus, the relationship between  $\ln N_t$  and  $t$  should be linear with slope  $\ln(\lambda)$  and  $y$ -intercept  $\ln N_0$ .

- (a) If  $\mathbf{X}$  represents a row of values, and  $\mathbf{Y}$  another row of values of the same size, the MATLAB<sup>®</sup> function `polyfit(X, Y, 1)` returns the slope  $m$  and  $y$ -intercept  $y_o$  of the line that best fits the data (in the sense of least squares regression) in  $\mathbf{X}$  and  $\mathbf{Y}$ :

$$y = mx + y_o.$$

Use this function to obtain estimates for the values of  $\ln N_0$  and  $\ln(\lambda)$

- (b) Obtain estimates for  $N_0$  and  $\lambda$ , and use them to generate a new set of predicted values for the US population. Plot these, along with the actual data, and assess how good the fit is.