Assignment #6

Due on Wednesday, October 10, 2007

Read Section 1.2 on *Nonlinear Models,* pp. 11–17, in Allman and Rhodes.

Read Section 1.3 on *Analyzing Nonlinear Models,* pp. 20–28 , in Allman and Rhodes.

Do the following problems

1. Problems 1.2.9 and 1.2.10 on page 18 in Allman and Rhodes.

2. Problems 1.3.1 and 1.3.2 on page 28 in Allman and Rhodes.

3. *(US Census Data, Revisited.)* In this problem and the next, we fit a logistic curve to the US Census data contained in the MS Excel file CensusDataUS.xls in the Math 36 website (`http://pages.pomona.edu/~ajr04747`)

   The idea for this fit is to observe that, if we write the logistic difference equation

   $$N_{t+1} = N_t + rN_t\left(1 - \frac{N_t}{K}\right)$$

   in the form $\dfrac{\Delta N}{N_t} = r - \dfrac{r}{K}N_t$, where $\Delta N = N_{t+1} - N_t$, then the logistic model predicts that the relationship between the *relative increments* $\dfrac{\Delta N}{N_t}$ and N_t should be linear with slope $-r/K$ and y–intercept r. Thus, the parameters r and K can be estimated from the data by a linear, least–squares regression fit of the relative increments versus the population density.

   (a) Use MATLAB®to define an array, `Y`, made up of the relative increments of the US population since census started being taken. The size of this new array should be one less than the size of the US population array.

   (b) Define an array, `N`, made up of the US population values up to the next to the last one (i.e., the census values from 1790 to 1990).

   (c) Plot `Y` versus `N`. Use the MATLAB®command `plot(N,Y,'k*')` and then type `hold on` in the command window in order to keep the plot.

   (d) Use the MATLAB®command `polyfit(N,Y,1)` to obtain the slope, m, and y–intercept, b, of the least–squares regression line of `Y` versus `N`, and sketch this line on the same graph obtained in the previous part.

   (e) Use the slope and y–intercept obtained in the previous part to estimate the intrinsic growth rate, r, and carrying capacity, K, for the US population.

4. *(US Census Data, Revisited (continued).)*

   (a) Use the estimates for r and K obtained in the previous problem, and the US population in 1790 as N_o, to compute population values predicted by the logistic model for each of the decades since 1790 until 2000. You may use the MATLAB®.m–file `LogisticK.m` to do these calculations

   (b) Plot the predicted and actual values on the same graph. How well do these predictions compare with the actual data? How does this fit compare with the Malthusian model fit of the data done in Problem (5) of Assignment #3?

5. Use the procedure outlined in the previous two problems to fit a logistic curve through the *Insect Population Values* data found in Table 1.6, p. 18, in Allman and Rhodes. What are the estimated values of r and K for the insect population?