Assignment #8

Due on Wednesday, October 31, 2007

Read Chapter 4 on *the continuous approach to modeling bacterial growth,* p. 29, in the class lecture notes webpage at `http://pages.pomona.edu/~ajr04747`

Do the following problems

1. Suppose a bacterial colony is growing according to the (continuous) Malthusian model. If the time, t, is measured in units of *division cycle* divided by $\ln 2$, give a formula for $N(t)$, given that $N(0) = N_o$. By how much does the population increase in one unit of time? (*Note:* A *division cycle* would correspond to the doubling time.)

2. Suppose a quantity Q varies in time according to the differential equation

$$\frac{dQ}{dt} = aQ + b, \tag{1}$$

where a and b are real constants with $a \neq 0$.

   (a) Use separation of variables to find the general solution to equation (1).

   (b) Find a solution to (1) satisfying $Q(0) = Q_o$.

   (c) What does equation (1) predict about $\lim_{t\to\infty} Q(t)$ if (i) $a < 0$, and (ii) if $a > 0$.

3. Give the equilibrium point of the differential equation (1) for $a \neq 0$. Discuss the stability type of this equilibrium point if (i) $a < 0$, and (ii) if $a > 0$. Sketch solutions starting near the equilibrium point for each of these cases.

A Conservation Principle for a One–Compartment Model. Suppose you are tracking the amount, $Q(t)$, of a substance in some predefined space or region, known as a *compartment,* at time t. (A compartment could represent, for instance, the bloodstream in the body of a patient, and $Q(t)$ the amount of a drug present in the bloodstream at time t). If we know, or can model, the rates at which the substance enters or leaves the compartment, then the rate of the change of the substance in the compartment is determined by the differential equation:

$$\frac{dQ}{dt} = \text{Rate of substance in} - \text{Rate of substance out}, \tag{2}$$

where we are assuming that Q is a differentiable function of time.

4. Assume that the rate at which a drug leaves the bloodstream and passes into the urine is proportional to the quantity of the drug in the blood at that time.

   (a) Use the conservation principle (2) to write down a differential equation for the quantity, Q, of the drug in the blood at time, t, in hours.

   (b) Solve the differential equation derived in part (a) for the case in which an initial dose of Q_o is injected directly into the blood.

   (c) Assume that 20% of the initial dose is left in the blood after 3 hours. Write a formula for computing $Q(t)$ for any time t, in hours.

   (d) What percentage of the initial dosage of the drug is left in the patient's body after 6 hours?

5. When people smoke, carbon monoxide is released into the air. Suppose that in a room of volume 60 m^3, air containing 5% carbon monoxide is introduced at a rate of 0.002 m^3/min. (This means that 5% of the volume of incoming air is carbon monoxide). Assume that the carbon monoxide mixes immediately with the air and the mixture leaves the room at the same rate as it enters.

   (a) Let $Q = Q(t)$ denote the volume (in cubic meters) of carbon monoxide in the room at any time t in minutes. Use the conservation principle (2) to write down a differential equation for Q.

   (b) Give the equilibrium solution, $\overline{Q}$, to the differential equation in part (a).

   (c) Solve the differential equation in part (a) under the assumption that the there is no carbon monoxide in the room initially, and sketch the solution.

   (d) Based on your solution to part (c), give the concentration, $c(t)$, of carbon monoxide in the room (in percent volume) at any time t in minutes. What happens to the value of $c(t)$ in the long run?

   (e) Medical texts warn that exposure to air containing 0.1% carbon monoxide for some time can lead to a coma. How many hours does it take for the concentration of carbon monoxide found in part (d) to reach this level?