Exam 2

December 7, 2007

Name: ____________________

Show all significant work and justify all your answers. This is a closed book exam. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 3 problems. Relax.

1. Suppose that the rate at which a drug leaves the bloodstream and passes into the urine at a given time is proportional to the quantity of the drug in the blood at that time.

   (a) Write down and solve a differential equation for the quantity, $Q = Q(t)$, of the drug in the blood at time, t, in hours. State all the assumptions you make and define all the parameters that you introduce.

   (b) Suppose that an initial dose of Q_o is injected directly into the blood, and that 20% of that initial amount is is left in the blood after 3 hours. Based on the solution you found in the previous part, write down $Q(t)$ for this situation and sketch its graph.

   (c) How much of the drug is left in the patient's body after 6 hours if the patient is given 100 mg initially?

2. The following equation models the evolution of a population that is being harvested at a constant rate:

$$\frac{dN}{dt} = 2N\left(1 - \frac{N}{200}\right) - 75$$

   (a) Give an interpretation for the model.

   (b) Find equilibrium points, determine the nature of their stability, and sketch a few possible solution curves.

   (c) According to model, what will happen if at time $t = 0$ the initial population density is 47? What do you conclude?

3. Luria and Delbrück[1] devised the following procedure (known as the *fluctuation test*) to estimate the *mutation rate, a*, for certain bacteria:

   Imagine that you start with a single normal bacterium (with no mutations) and allow it to grow to produce several bacteria. Place each of these bacteria in test–tubes each with media conducive to growth. Suppose the bacteria in the test–tubes are allowed to reproduce for n division cycles. After the n^{th} division cycle, the content of each test–tube is placed onto a agar plate containing a virus population which is lethal to the bacteria which have not developed resistance. Those bacteria which have mutated into resistant strains will continue to replicate, while those that are sensitive to the virus will die. After certain time, the resistant bacteria will develop visible colonies on the plates. The number of these colonies will then correspond to the number of resistant cells in each test tube at the time they were exposed to the virus.

   (a) Estimate the probability, p_o, that at the end of the n division cycles there will be no resistant bacteria. State all assumptions you make and justify your answer.

   (b) In one of the experiments of Luria and Delbrück in 1943, they observed that out of 100 cultures, each of about 2.8×10^8 bacteria, 57 showed no resistant bacteria. Use this information to estimate:

      i. The average number of mutations, μ, that occurred before the bacteria were exposed to the virus;

      ii. The mutation rate, a; that is, the probability that a given bacterium will mutate in a division cycle.

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[1] (1943) *Mutations of bacteria from virus sensitivity to virus resistance. Genetics*, **28**, 491–511