Assignment #11

Due on Monday, October 13, 2008

Read Section 7.4 on *The Derivative,* pp. 187–197, in Bressoud.

Read Section 3.1 on *The Calculus of Curves,* pp. 53–65, in Bressoud.

Background and Definitions
Let I denote an open interval of real numbers, and let $\sigma\colon I \to \mathbb{R}^n$ be a path in $\mathbb{R}^n$. If σ is differentiable at $t \in I$, then

$$\sigma(t+h) = \sigma(t) + h\mathbf{v}(t) + E_t(h), \text{ where } \lim_{h\to 0} \frac{\|E_t(h)\|}{|h|} = 0.$$

If σ is differentiable at every $t \in I$, the vector valued function $\mathbf{v}(t)$ is called the *velocity* of the path and is denoted by $\sigma'(t)$ for all $t \in I$.

Do the following problems

1. Let I denote an open interval in $\mathbb{R}$. Suppose that $\sigma\colon I \to \mathbb{R}^n$ and $\gamma\colon I \to \mathbb{R}^n$ are paths in $\mathbb{R}^n$. Define a real valued function $f\colon I \to \mathbb{R}$ of a single variable by
$$f(t) = \sigma(t) \cdot \gamma(t) \quad \text{for all } \ t \in I;$$
that is, $f(t)$ is the dot product of the two paths at t.
Show that if σ and γ are both differentiable on I, then so is f, and
$$f'(t) = \sigma'(t) \cdot \gamma(t) + \sigma(t) \cdot \gamma'(t) \quad \text{for all } \ t \in I.$$

2. Let $\sigma\colon I \to \mathbb{R}^n$ denote a differentiable path in $\mathbb{R}^n$. Show that if $\|\sigma(t)\|$ is constant for all $t \in I$, then $\sigma'(t)$ is orthogonal to $\sigma(t)$ for all $t \in I$.

3. Exercise 14 on page 66 in the text.

4. A particle is following a path in three–dimensional space given by
$$\sigma(t) = (e^t, e^{-t}, 1-t) \quad \text{for } \ t \in \mathbb{R}.$$
At time $t_o = 1$, the particle flies off on a tangent.
   (a) Where will the particle be at time $t_1 = 2$?
   (b) Will the particle ever hit the xy–plane? Is so, find the location on the xy plane where the particle hits.

5. Suppose the velocity, $\sigma'(t)$, of a path $\sigma\colon I \to \mathbb{R}^n$ is itself differentiable. We denote its derivative by $\sigma''(t)$ and say that σ is *twice–differentiable.* What can you say about a twice–differentiable path, σ, for which $\sigma''(t)$ is the zero vector in $\mathbb{R}^n$ for all $t \in I$?