

Assignment #17

Due on Monday, November 10, 2008

Read Section 3.1 on *The Calculus of Curves*, pp. 53–65, in Bressoud.

Read Section 5.2 on *Line Integrals*, pp. 113–119, in Bressoud.

Do the following problems

1. Exercise 4 on page 119 in the text.
2. Exercises 6(d)(e)(f) on pages 119 and 120 in the text.
3. Let $\sigma: [a, b] \rightarrow \mathbb{R}^n$ be a C^1 parametrization of a curve C in \mathbb{R}^n . Let $h: [c, d] \rightarrow [a, b]$ be a one-to-one and onto map such that $h'(t) > 0$ for all $t \in [c, d]$. Define

$$\gamma(t) = \sigma(h(t)) \quad \text{for all } t \in [c, d].$$

$\gamma: [c, d] \rightarrow \mathbb{R}^n$ is called a *reparametrization* of σ .

Let $F: U \rightarrow \mathbb{R}^n$ denote a continuous vector field defined on a region U of \mathbb{R}^n which contains the curve C . Show that

$$\int_a^b F(\sigma(\tau)) \cdot \sigma'(\tau) \, d\tau = \int_c^d F(\gamma(t)) \cdot \gamma'(t) \, dt.$$

Thus, the line integral

$$\int_C F \cdot T \, ds$$

is independent of reparametrization.

4. Let $\sigma: [0, 1] \rightarrow \mathbb{R}^n$ be a C^1 parametrization of a curve C in \mathbb{R}^n . Give a C^1 reparametrization, $\gamma: [0, 1] \rightarrow \mathbb{R}^n$, of σ in which the curve C is traversed in the opposite direction as that of σ . What is γ' in terms of σ' ?
5. Recall that the flux of a 2-dimensional vector field,

$$F(x, y) = P(x, y) \hat{i} + Q(x, y) \hat{j},$$

across a simple, C^1 , closed curve, C , is given by

$$\int_C P \, dy - Q \, dx.$$

Compute the flux of the following fields across the given curves

- (a) $F(x, y) = x^2 \hat{i} + y^2 \hat{j}$ and C is the boundary of the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$ and $(0, 1)$.
- (b) $F(x, y) = x \hat{i} + y \hat{j}$ and C is the boundary of the unit circle