

## Assignment #8

Due on Friday October 3, 2008

Read Section 7.4 on *The Derivative*, pp. 187–197, in Bressoud.

Do the following problems

1. Let  $f$  denote a real valued function defined on some open interval around  $a \in \mathbb{R}$ . Consider a line of slope  $m$  and equation

$$L(x) = f(a) + m(x - a) \quad \text{for all } x \in \mathbb{R}.$$

Suppose that this line is the best approximation to the function  $f$  at  $a$  in the sense that

$$\lim_{x \rightarrow a} \frac{|E(x)|}{|x - a|} = 0,$$

where  $E(x) = f(x) - L(x)$  for all  $x$  in the interval in which  $f$  is defined.

Prove that  $f$  is differentiable at  $a$ , and that  $f'(a) = m$ .

2. Recall that a function  $F: U \rightarrow \mathbb{R}^m$ , where  $U$  is an open subset for  $\mathbb{R}^n$ , is said to be differentiable at  $x \in U$  if and only if there exists a unique linear transformation  $T_x: \mathbb{R}^n \rightarrow \mathbb{R}^m$  such that

$$\lim_{\|y-x\| \rightarrow 0} \frac{\|F(y) - F(x) - T_x(y-x)\|}{\|y-x\|} = 0.$$

Prove that if  $F$  is differentiable at  $x$ , then it is also continuous at  $x$ .

Give an example that shows that the converse of this assertion is not true

3. Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be defined by  $f(x, y) = \sqrt{|xy|}$  for all  $(x, y) \in \mathbb{R}^2$ . Show that  $f$  is not differentiable at  $(0, 0)$ .
4. Exercise 4 on page 197 in the text.
5. Exercise 6 on page 197 in the text.