Exam 1 (Part I)

Wednesday, October 15, 2008

Name: ______________________________

This is a closed book exam. Show all significant work and justify all your answers. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 3 problems. Relax.

1. The points $P(1,0,0)$, $Q(0,4,0)$ and $R(0,0,7)$ determine a unique plane in three dimensional Euclidean space, $\mathbb{R}^3$.

   (a) Give the equation of the plane.

   (b) Find the (shortest) distance from the plane to the origin in $\mathbb{R}^3$ and give the coordinates of the point on the plane which is the closest to the origin.

2. Let D denote an open subset of the xy–plane, $\mathbb{R}^2$, and let $f\colon D \to \mathbb{R}$ be a scalar filed defined on D.

   (a) State precisely what it means for f to be continuous at $(x_o, y_o) \in D$.

   (b) Let $f(x,y) = xy$ for all $(x,y) \in \mathbb{R}^2$. Use the inequality

   $$ab \leqslant \frac{1}{2}(a^2 + b^2) \quad \text{for all nonnegative real numbers } a, b,$$

   and the Squeeze Theorem to prove that f is continuous at the origin $(0,0)$.

3. Let U denote an open subset of $\mathbb{R}^n$, and let $F\colon U \to \mathbb{R}^m$ be a vector field on U.

   (a) State precisely what it means for F to be differentiable at $u \in U$.

   (b) Suppose that a vector field $F\colon \mathbb{R}^n \to \mathbb{R}^m$ is a linear map. Prove that F is differentiable at every $u \in U$, and compute its derivative map

   $$DF(u)\colon \mathbb{R}^n \to \mathbb{R}^m$$

   at u, for all $u \in \mathbb{R}^n$.