Assignment #10

Due on Monday, October 26, 2009

Read Section 7.4 on *The Derivative,* pp. 187–197, in Bressoud.

Read Section 7.6 on *The Chain Rule,* pp. 201–205, in Bressoud.

Do the following problems

1. Let $U = \mathbb{R}^n \backslash \{\mathbf{0}\} = \{v \in \mathbb{R}^n \mid v \neq \mathbf{0}\}$ and define $f\colon \mathbb{R}^n \to \mathbb{R}$ by

$$f(v) = \|v\| \quad \text{for all} \quad v \in \mathbb{R}.$$

   (a) Prove that f is differentiable on U.

   (b) Prove that f is not differentiable at the origin in $\mathbb{R}^n$.

2. Let I be and open interval of real numbers, and suppose that $\sigma\colon I \to \mathbb{R}^n$ is a differentiable path satisfying $\sigma(t) \neq \mathbf{0}$ for all $t \in I$. Show that the function $g\colon I \to \mathbb{R}$ defined by $g(t) = \|\sigma(t)\|$ for all $t \in I$ is differentiable on I and compute its derivative.

3. Let I be an open interval of real numbers and U ba an open subset of $\mathbb{R}^n$. Suppose that $\sigma\colon I \to \mathbb{R}^n$ is a differentiable path and that $f\colon U \to \mathbb{R}$ is a differentiable scalar field. Assume also that the image of I under σ, $\sigma(I)$, is contained in U. Suppose also that the derivative of the path σ satisfies

$$\sigma'(t) = -\nabla f(\sigma(t)) \quad \text{for all} \quad t \in I.$$

   Show that if the gradient of f along the path σ is never zero, then f decreases along the path as t increases.

   Suggestion: Use the Chain Rule to compute the derivative of $f(\sigma(t))$.

4. A set $U \subseteq \mathbb{R}^n$ is said to be **path connected** iff for any vectors x and y in U, there exists a differentiable path $\sigma\colon [0,1] \to \mathbb{R}^n$ such that $\sigma(0) = x$, $\sigma(1) = y$ and $\sigma(t) \in U$ for all $t \in [0,1]$; i.e., any two elements in U can be connected by a differentiable path whose image is entirely contained in U.

   Suppose that U is an open, path connected subset of $\mathbb{R}^n$. Let $f\colon U \to \mathbb{R}$ be a differentiable scalar field such that $\nabla f(x)$ is the zero vector for all $x \in U$. Prove that f must be constant.

5. Exercises 2 and 4 on page 207 in the text.