Assignment #13

Due on Wednesday, November 18, 2009

Read Section 3.1 on *The Calculus of Curves,* pp. 53–65, in Bressoud.

Read Section 5.2 on *Line Integrals,* pp. 113–119, in Bressoud.

Do the following problems

1. Consider a portion of a helix, C, parametrized by the path
$$\sigma(t) = (\cos t, t, \sin t) \quad \text{for} \quad 0 \leqslant t \leqslant \pi.$$
Let $F(x, y, z) = x\ \widehat{i} + y\ \widehat{j} + z\ \widehat{k}$, for all $(x, y, z) \in \mathbb{R}^3$, be a vector field in $\mathbb{R}^3$. Evaluate the line integral $\displaystyle\int_C F \cdot T\ \mathrm{d}s$; that is, the integral of the tangential component of the field F along the curve C.

2. Evaluate
$$\int_C yz\ \mathrm{d}x + xz\ \mathrm{d}y + xy\ \mathrm{d}z$$
where C is the directed line segment from the point $(1, 1, 0)$ to the point $(3, 2, 1)$ in $\mathbb{R}^3$.

3. Exercises 1(a)(b)(c) on page 119 in the text.

4. Exercises 1(d)(e)(f) on page 119 in the text.

5. Let $f : U \to \mathbb{R}$ be a C^1 scalar field defined on an open subset U of $\mathbb{R}^n$. Define the vector field $F : U \to \mathbb{R}^n$ by $F(x) = \nabla f(x)$ for all $x \in U$. Suppose that C is a C^1 simple curve in U connecting the point x to the point y in U. Show that
$$\int_C F \cdot T = f(y) - f(x).$$
Conclude therefore that the line integral of F along a path from x to y in U is independent of the path connecting x to y. The field F is called a *gradient field.*