Assignment #8

Due on Wednesday, October 14, 2009

Read Section 7.4 on *The Derivative,* pp. 187–197, in Bressoud.

Do the following problems

1. Let f denote a real valued function defined on some open interval around $a \in \mathbb{R}$. Consider a line of slope m and equation

$$L(x) = f(a) + m(x - a) \quad \text{for all} \quad x \in \mathbb{R}.$$

Suppose that this line if the best approximation to the function f at a in the sense that

$$\lim_{x \to a} \frac{|E(x)|}{|x - a|} = 0,$$

where $E(x) = f(x) - L(x)$ for all x in the interval in which f is defined.

Prove that f is differentiable at a, and that $f'(a) = m$.

2. Recall that a function $F \colon U \to \mathbb{R}^m$, where U is an open subset for $\mathbb{R}^n$, is said to be differentiable at $u \in U$ if and only if there exists a unique linear transformation $T_u \colon \mathbb{R}^n \to \mathbb{R}^m$ such that

$$\lim_{\|v-u\| \to 0} \frac{\|F(v) - F(u) - T_u(y - x)\|}{\|v - u\|} = 0.$$

Prove that if F is differentiable at u, then it is also continuous at u.

Give an example that shows that the converse of this assertion is not true

3. Let $f \colon \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = \sqrt{|xy|}$ for all $(x, y) \in \mathbb{R}^2$. Show that f is not differentiable at $(0, 0)$.

4. Exercise 4 on page 197 in the text.

5. Exercise 6 on page 197 in the text.