Assignment #12

Due on Friday, November 13, 2009

Read Section 6.3 on *Maximum Likelihood Tests,* pp. 333–339, in Hogg, Craig and McKean.

Do the following problems

1. Suppose that you observe n iid Bernoulli(p) random variables, denoted by $X_1, X_2, \ldots, X_n$. Find the LRT rejection region for the test of $\text{H}_o\colon p \leqslant p_o$ versus $\text{H}_1\colon p > p_o$ in terms of the test statistic $Y = \sum_{i=1}^{n} X_i$.

2. Consider the likelihood ratio test for $\text{H}_o\colon\ p = p_o$ versus $\text{H}_1\colon\ p = p_1$, where $p_o \neq p_1$, based on a random sample $X_1, X_2, \ldots, X_n$ from a Bernoulli(p) distribution for $0 < p < 1$. Show that, if $p_1 > p_o$, then the likelihood ratio statistic for the test is is a monotonically decreasing function of $Y = \sum_{i=1}^{n} X_i$. . Conclude, therefore, that if the test rejects H_o at the significance level α for an observed value $\widehat{y}$ of Y, it will also rejects H_o at that level for $Y > \widehat{y}$.

3. We wish to use an LRT to test the hypothesis $\text{H}_o\colon\ \mu = \mu_o$ against the alternative $\text{H}_1\colon\ \mu \neq \mu_o$ based on a random sample, $X_1, X_2, \ldots, X_n$, from a normal$(\mu, 1)$ distribution.

   (a) Give the maximum likelihood estimator, $\widehat{\mu}$, for μ based on the sample.

   (b) Give the likelihood ratio statistic for the test.

   (c) Express the LRT rejection region in terms of the sample mean $\overline{X}_n$.

4. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a uniform$(0, \theta)$ distribution for some parameter $\theta > 0$.

   (a) Give the likelihood function $L(\theta \mid x_1, x_2, \ldots, x_n)$.

   (b) Give the maximum likelihood estimator for θ.

5. Let R denote the rejection region for an LRT of $\text{H}_o\colon\ \theta = \theta_o$ versus $\text{H}_1\colon\ \theta = \theta_1$ based on a random sample, $X_1, X_2, \ldots, X_n$, from continuous distribution with pdf $f(x \mid \theta)$. Let $L(\theta \mid x_1, x_2, \ldots, x_n)$ denote the likelihood function. Suppose the LRT has significance level α.

   (a) Explain why
   $$\alpha = \int_R L(\theta_o \mid x_1, x_2, \ldots, x_n)\ \mathrm{d}x_1\ \mathrm{d}x_2 \cdots\ \mathrm{d}x_n.$$

   (b) Explain why the power of the test is
   $$\gamma(\theta_1) = \int_R L(\theta_1 \mid x_1, x_2, \ldots, x_n)\ \mathrm{d}x_1\ \mathrm{d}x_2 \cdots\ \mathrm{d}x_n.$$

   (c) Explain why
   $$\alpha \leqslant c\gamma(\theta_1),$$
   where c is the critical value used in the definition of the rejection region, R, for the LRT.