Assignment #2

Due on Monday, September 14, 2009

Read Section 5.1 on *Sampling and Statistics,* pp. 233–236, in Hogg, Craig and McKean.

Do the following problems

1. The reason that the function $M_X(t)$ is called the moment generating function for random variable X is that the n^{th} derivative of $M_X(t)$ at $t = 0$ is $E(X^n)$, the n^{th} moment of the random variable X; that is,

$$M_X^{(n)}(0) = E(X^n) \quad \text{for} \ \ n = 1, 2, 3, \ldots \tag{1}$$

   (a) Verify (1) for the case in which X is continuous with pdf f_X. What assumptions do you need to make about the mgf in your derivation?

   (b) Show that if the mgf of X exists on some interval around 0, then

$$\text{var}(X) = M_X''(0) - \left[M_X'(0)\right]^2$$

2. Let $\lambda > 0$. A random variable X is said to follow a Poisson(λ) distribution if X takes the values $0, 1, 2, 3, \ldots$ and the pmf of X is given by

$$p_X(k) = \frac{\lambda^k}{k!}\, e^{-\lambda} \quad \text{for all} \ \ k = 0, 1, 2, 3, \ldots$$

   Compute the mgf of a Poisson(λ) random variable, X. For which values of t is the mgf defined?

3. Use the result of Problem 2 to compute the mean and variance of a Poisson(λ) distribution. What do you discover?

4. Let $X_1, X_2, \ldots, X_n$ be a random sample from a Poisson(λ) distribution. Define $Y_n = X_1 + X_2 + \cdots + X_n$. Give the sampling distribution for Y_n. What do you discover?

5. Let $X_1, X_2, X_3 \ldots$ be a sequence of random variable satisfying $X_n \sim \text{binomial}(n,p)$ for all n. Assume also that $np = \lambda$, where λ is a fixed parameter.

   Compute $M_{X_n}(t)$ for all n and determine the limit

   $$\lim_{n\to\infty} M_{X_n}(t).$$

   What do you discover?

   Hint: Observe that $p = \dfrac{\lambda}{n} \to 0$ as $n \to \infty$ since λ is assumed to be fixed.