Assignment #7

Due on Monday, October 12, 2009

Read Section 5.3 on *More on Confidence Intervals,* pp. 254–260, in Hogg, Craig and McKean.

Do the following problems

1. Assume that a random variable, T, has a t distribution with n degrees of freedom. Define $X = T^2$. Determine the distribution of X.

2. Recall that in Problem 3 of Assignment #4 you verified that if X and Y are independent random variables with pdfs $f_{_X}$ and $f_{_Y}$, respectively, and $W = Y/X$, then the pdf of W is given by
$$f_{_W}(w) = \int_{-\infty}^{\infty} |u| f_{_X}(u) f_{_Y}(wu)\, \mathrm{d}u. \tag{1}$$
Suppose that X and Y are independent exponential(1) random variables and define $W = Y/X$. Compute the pdf of W and determine the type of distribution that W has.

3. Let $X \sim \chi^2(n-1)$ and $Y \sim \chi^2(m-1)$ be independent random variables and define $W = \dfrac{Y/(m-1)}{X/(n-1)}$. Use the formula in (1) to compute the pdf of W. Determine the type of distribution that W has.

4. Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal$(\mu_{_X}, \sigma^2)$ distribution and $Y_1, Y_2, \ldots, Y_m$ be a random sample from a normal$(\mu_{_Y}, \sigma^2)$. Let $S^2_{_X}$ denote the sample variance of the random sample $X_1, X_2, \ldots, X_n$ and $S^2_{_Y}$ that of the random sample $Y_1, Y_2, \ldots, Y_m$. Determine the distribution of $S^2_{_Y}/S^2_{_X}$ and use that information to show how to find $\mathrm{P}\left(\dfrac{S^2_{_Y}}{S^2_{_X}} > c\right)$ for any $c > 0$.

5. Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal$(\mu_{_X}, \sigma^2_{_X})$ distribution and $Y_1, Y_2, \ldots, Y_m$ be a random sample from a normal$(\mu_{_Y}, \sigma^2_{_Y})$. Let $S^2_{_X}$ denote the sample variance of the random sample $X_1, X_2, \ldots, X_n$ and $S^2_{_Y}$ that of the random sample $Y_1, Y_2, \ldots, Y_m$. Determine the distribution of $\dfrac{S^2_{_Y}/\sigma^2_{_Y}}{S^2_{_X}/\sigma^2_{_X}}$ and use that information to explain how to find a 95% confidence interval for $\sigma^2_{_Y}/\sigma^2_{_X}$.