Some Common Distributions

(I) Discrete Distributions

(1) *The Bernoulli Distribution*

$X \sim \text{Bernoulli}(p)$ for $0 < p < 1$

pmf: $p_X(x) = p^x(1-p)^{1-x}$ for $x = 0, 1$

Expected value: $E(X) = p$

Variance: $\text{Var}(X) = p(1-p)$

mgf: $M_X(t) = pe^t + 1 - p$ for $t \in \mathbf{R}$

(2) *The Binomial Distribution*

$X \sim \text{binomial}(p, n)$ for $0 < p < 1$, $n = 2, 3, 4, \ldots$

pmf: $p_X(x) = \binom{n}{x} p^x(1-p)^{n-x}$ for $x = 0, 1, 2, \ldots, n$

Expected value: $E(X) = np$

Variance: $\text{Var}(X) = np(1-p)$

mgf: $M_X(t) = (pe^t + 1 - p)^n$ for $t \in \mathbf{R}$

(3) *The Geometric Distribution*

$X \sim \text{geometric}(p)$ for $0 < p < 1$

pmf: $p_X(x) = p(1-p)^{x-1}$ for $x = 1, 2, 3, \ldots$

Expected value: $E(X) = \dfrac{1}{p}$

Variance: $\text{Var}(X) = \dfrac{1-p}{p^2}$

mgf: $M_X(t) = \dfrac{p}{e^{-t} + p - 1}$ for $t < \ln\left(\dfrac{1}{1-p}\right)$

(4) *The Poisson Distribution*

$X \sim \text{Poisson}(\lambda)$ for $\lambda > 0$

pmf: $p_X(x) = \dfrac{\lambda^x}{x!} e^{-\lambda}$ for $x = 0, 1, 2, 3, \ldots$

Expected value: $E(X) = \lambda$

Variance: $\text{Var}(X) = \lambda$

mgf: $M_X(t) = e^{\lambda(e^t - 1)}$ for $t \in \mathbf{R}$

(5) *The Discrete Uniform Distribution*

$X \sim$ discrete uniform(N) for $N = 1, 2, 3, \ldots$

pmf: $p_X(x) = \dfrac{1}{N}$ for $x = 0, 1, 2, \ldots, N$

Expected value: $E(X) = \dfrac{N+1}{2}$

Variance: $\text{Var}(X) = \dfrac{(N+1)(N-1)}{12}$

mgf: $M_X(t) = \dfrac{1}{N}\displaystyle\sum_{i=1}^{N} e^{it}$ for $t \in \mathbf{R}$

(6) *The Hypergeometric Distribution*

$X \sim$ hypergeometric(N, M, K) for $N, M, K > 0$ with $K < M$ and $M < N$.

pmf: $p_X(x) = \dfrac{\binom{M}{x}\binom{N-M}{K-x}}{\binom{N}{K}}$ for $x = 0, 1, 2, \ldots, K$

Expected value: $E(X) = \dfrac{KM}{N}$

Variance: $\text{Var}(X) = \dfrac{KM}{N}\dfrac{(N-M)(N-K)}{N(N-1)}$

(II) **Continuous Distributions**

(1) *The Uniform Distribution*

$X \sim$ uniform(a, b) for $a < b$

pdf: $f_X(x) = \dfrac{1}{b-a}$ for $a \leq x \leq b$ and 0 elsewhere

Expected value: $E(X) = \dfrac{a+b}{2}$

Variance: $\text{Var}(X) = \dfrac{(b-a)^2}{12}$

mgf: $M_X(t) = \dfrac{e^{bt} - e^{at}}{(b-a)t}$ for $t \neq 0$ and $M_X(0) = 1$

(2) *The Exponential Distribution*

$X \sim$ exponential(β) for $\beta > 0$

pdf: $f_X(x) = \dfrac{1}{\beta}\, e^{-x/\beta}$ for $0 \leq x < \infty$ and 0 elsewhere

Expected value: $E(X) = \beta$

Variance: $\text{Var}(X) = \beta^2$

mgf: $M_X(t) = \dfrac{1}{1-\beta t}$ for $t < \dfrac{1}{\beta}$

(3) *The Cauchy Distribution*

$X \sim \text{Cauchy}(\theta, \sigma)$ for $\sigma > 0$ and $-\infty < \theta < +\infty$

pdf: $f_X(x) = \dfrac{1/(\pi\sigma)}{1 + [(x-\theta)/\sigma]^2}$ for $-\infty < x < +\infty$

Expected value: does not exist

Variance: does not exist

mgf: does not exist

(4) *The Normal Distribution*

$X \sim \text{normal}(\mu, \sigma^2)$ for $-\infty < \mu < \infty$ and $\sigma > 0$.

pdf: $f_X(x) = \dfrac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-(x-\mu)^2/(2\sigma^2)}$ for $-\infty < x < +\infty$

Expected value: $E(X) = \mu$

Variance: $\text{Var}(X) = \sigma^2$

mgf: $M_X(t) = e^{\mu t + \sigma^2 t^2/2}$ for $t \in \mathbf{R}$

(5) *The Gamma Distribution*

$X \sim \text{Gamma}(\alpha, \beta)$ for $\alpha, \beta > 0$

pdf: $f_X(x) = \dfrac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$ for $0 < x < \infty$ and zero elsewhere; where Γ is the *gamma function* defined by

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} \text{d}t \quad \text{for all real values of } z \text{ except } 0, -1, -2, -3, \ldots$$

Expected value: $E(X) = \alpha\beta$

Variance: $\text{Var}(X) = \alpha\beta^2$

mgf: $M_X(t) = \left(\dfrac{1}{1-\beta t}\right)^\alpha$ for $t < \dfrac{1}{\beta}$

(6) *The Chi Squared Distribution with p degrees of freedom*

$X \sim \chi^2(p)$ for $p = 1, 2, 3, \ldots$

pdf: $f_X(x) = \dfrac{1}{\Gamma(p/2)2^{p/2}} x^{(p/2)-1} e^{-x/2}$ for $0 < x < \infty$

and zero elsewhere

Expected value: $E(X) = p$

Variance: $\text{Var}(X) = 2p$

mgf: $M_X(t) = \left(\dfrac{1}{1-2t}\right)^{p/2}$ for $t < \dfrac{1}{2}$

(7) *The t Distribution with r degrees of freedom*

$X \sim t(r)$ for $r = 1, 2, 3, \ldots$

pdf: $$f_X(x) = \frac{\Gamma((r+1)/2)}{\Gamma(r/2)} \frac{1}{\sqrt{r\pi}} \frac{1}{(1+(x^2/r))^{(r+1)/2}} \quad -\infty < x < \infty$$

Expected value: $E(X) = 0$ if $r > 1$

Variance: $\text{Var}(X) = \dfrac{r}{r-2}$ if $r > 2$

(8) *The F Distribution with $(\nu_1 \nu_2)$ degrees of freedom*

$X \sim F(\nu_1, \nu_2)$ for $\nu_1, \nu_2 = 1, 2, 3, \ldots$

pdf: $$f_X(x) = \frac{\Gamma((\nu_1+\nu_2)/2)}{\Gamma(\nu_1/2)\Gamma(\nu_2/2)} \left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2} \frac{x^{(\nu_1-2)/2}}{(1+(\nu_1 x/\nu_2))^{(\nu_1+\nu_2)/2}}$$

if $0 \leq x < \infty$ and zero elsewhere

Expected value: $E(X) = \dfrac{\nu_2}{\nu_2 - 2}$ if $\nu_2 > 2$

Variance: $\text{Var}(X) = 2\left(\dfrac{\nu_2}{\nu_2-2}\right)^2 \dfrac{(\nu_1+\nu_2-2)}{\nu_1(\nu_2-4)}$ if $\nu_2 > 4$