Assignment #19

Due on Monday, November 21, 2011

Read Section 5.2, *The Principle of Linearized Stability,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`, starting on page 80.

Do the following problems

1. Consider the first order differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}t} = 5 - 6y + y^2$.

   (a) Find all equilibrium solutions of the equation, and determine the nature of their stability.

   (b) Sketch at least two possible solutions for the equation.

   (c) For $y(0) = 4$, sketch the solution, $y(t)$, and compute $\lim_{t\to\infty} y(t)$.

2. The following equation models the evolution of a population that is being harvested at a constant rate:

$$\frac{dN}{dt} = 2N - 0.01N^2 - 75.$$

   Find equilibrium solutions and sketch a few possible solution curves. According to model, what will happen if at time $t = 0$ the initial population densities are 40, 60, 150, or 170?

3. Consider the modified logistic model

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)\left(\frac{N}{T} - 1\right)$$

   where $N(t)$ denotes the population density at time t, $r > 0$ and $0 < T < K$.

   (a) Find the equilibrium solutions and determine the nature of their stability.

   (b) Sketch other possible solutions to the equation.

   (c) Describe what the model predicts about the population and give a possible explanation.

4. *[Harvesting]* The following differential equation models the growth of a population of size $N = N(t)$ that is being harvested at a rate proportional to the population density
$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - EN, \tag{1}$$
where r, K and E are non–negative parameters with $r > 0$ and $K > 0$.

   (a) Give an interpretation for this model. In particular, give interpretation for the term EN. The parameter E is usually called the harvesting *effort.*

   (b) Calculate the equilibrium points for the equation (1), and give conditions on the parameters that yield a biologically meaningful equilibrium point. Determine the nature of the stability of that equilibrium point. Sketch possible solutions to the equation in this situation.

   (c) What does the model predict if $E \geqslant r$?

5. *[Harvesting, continued]* Suppose that $0 < E < r$ in equation (1), and let $\overline{N}$ denote the positive equilibrium point. The quantity $Y = E\overline{N}$ is called the *harvesting yield.*

   (a) Find the value of E for which the harvesting yield is the largest possible; this value of the yield is called the *maximum sustainable yield.*

   (b) What is the value of the equilibrium point for which there is the maximum sustainable yield?