Assignment #4

Due on Friday, September 16, 2011

Read Section 4.2 on *Properties of Definite Integrals,* pp. 144–150, in *Essential Calculus with Applications* by Richard A. Silverman.

Read Chapter 4, *Applications of Integral Calculus,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`, starting on page 17.

Do the following problems

1. Solve the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = t\sin(t^2); \\ y(0) = 0, \end{cases}$$

for $t \in \mathbf{R}$.

2. Solve the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = \dfrac{\cos(\pi + \sqrt{t})}{\sqrt{t}}; \\ y(\pi^2) = 1, \end{cases}$$

for $t \geqslant 0$.

3. Let the graph of $y = f(t)$ be as sketched in Figure 1 on page 2 and put

$$F(t) = \int_0^t f(\tau)\ d\tau, \text{ for } t \geqslant 0.$$

   (a) Based on the sketch in Figure 1, determine intervals on which (i) $F(t)$ increases, (ii) $F(t)$ decreases, (iii) the graph of $y = F(t)$ is concave up, and (iv) the graph of $y = F(t)$ is concave down.

   (b) Estimate the times at which $F(t)$ is (i) a local maximum, and (ii) (i) a local minimum.

   (c) Locate any inflection points in the graph of $y = F(t)$

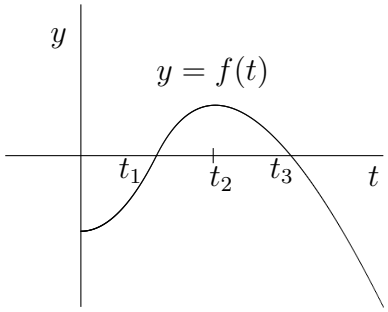

Figure 1: Sketch of graph of $y = f(t)$

4. Let f and F be as in Problem 3. Use the qualitative information obtained in Problem 3 to sketch the graph of $y = F(t)$.

5. Let f and F be as in Problem 3. Given that $t_2 = 2$ and t_3 is about 3 and $f(t_2)$ is about 0.75, estimate the maximum value of F over the range of values of t pictured in Figure 1.

   Suggestion: For this problem you may also assume the estimates $f(0) \approx -1$ and $t_1 \approx 1$.